공학석사학위논문

새로운 보정 척도 확률잔차와 확률잔차를 적용한 저널베어링 회전체 시스템 동특성 모델의 통계적 검증

A New Calibration Metric – Probability Residual (PR) and Its Validation Practice for Rotor Dynamics Model of a Journal Bearing Rotor System

2016 년 8 월

서울대학교 대학원

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최환오의 공학석사 학위논문을 인준함

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Abstract

A New Calibration Metric - Probability Residual (PR) and Its Validation Practice for Rotor Dynamics Model of a Journal Bearing Rotor System

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In constructing the computational model of engineered systems such as a journal bearing rotor systems, statistical model calibration method is often used since the statistical model emulates the actual behavior of the engineered systems with uncertainties. A calibration metric, which quantifies the degree of agreement or disagreement between computational and experimental results, is one of the key components in the statistical model calibration. However, some existing calibration metrics such as log-likelihood and Kullback-Leibler divergence (KLD) have limitations in constructing an accurate computational model. To overcome this problems, this study proposes a new calibration metric, probability residual (PR). The PR metric is defined as the sum of the product of scale factor and square of residuals. The scale factor scales the PDF in specific range, which enables to improve the calibration efficiency. The square of residuals makes the PR a convex form, which guarantees existence of global optimum. So as to evaluate the performance of the PR metric, this study uses mathematical models and employs statistical models of the journal bearing rotor system appropriate to normal and rubbing state. As a result, the PR metric performed better than other metrics including log-likelihood and KLD in terms of the calibration accuracy and efficiency, and the calibrated journal bearing rotor model with PR was proved in valid by the hypothesis testing. In summary, the proposed PR metric is promising to be applied in building an accurate computational model.

Keywords: Statistical Model Validation
Statistical Model Calibration
Calibration Metric
Validity Check
Journal Bearing Rotor System
Fault diagnosis
Hierarchical Frame Work

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Nomenclatures

x_i	experimental point at i th different validation site
y^e	empirical PDF value
$F_{x_i}^m$	marginal CDF of x_i
u_i	u values in universal probability scale at i th different validation site
Z_L	log-likelihood
E_i	experimental data
θ	statistical parameters vector
P_C	PDF of the computational response
P_E	PDF of the experimental data
f_E	continuous function of experiment
f_C	continuous function of computational response
$Z_{KLD}(\Vert)$	original form of KLD
$Z_{KLD}(,)$	symmetric form of KLD
S	scale factor
C	arbitrary constant controlling the scale factor
X	random input variables
Y	response
μ_x	mean of exact solution

σ_x	standard deviation of exact solution
$\overline{\mu}_x$	mean of the approximated solution
$\overline{\sigma}_x$	standard deviation of the approximated solution
$H_{Crestfactor}$	crest factor
x_{rms}	root mean squares of the signal
x_{peak}	peak length of the signal

Chapter 1. Introduction

1.1 Background and Motivation

In order to diagnose an engineered system such as a journal bearing rotor system, the abnormal data of the system is essential since the fault diagnosis is conducted based on the abnormal data. If there is no abnormal data of an operating system, it can be obtained from the experiment by seeding anomaly on testbed. But proceeding experiments under harsh conditions sometimes causes expensive cost and it has difficulties in realization, so that only few abnormal data can be obtained. In this case, it can be an alternative to use an artificial data obtained from the computational model imitating the real engineered system, because the computational model is relatively free to previous constraints and it can reflect the law of physics in the engineered system. If the computational model is used to complement the scarcity of the experimental data, the ability to emulate the actual behavior of the engineered systems comes to be significant.

Many of the existing computational models adopt the deterministic method. But the physical phenomenon in engineered fields such as the journal bearing rotor system appears not into deterministic values but into statistical values because of the uncertainties of nature. [1-3] For this reason, the existing deterministic model which receives deterministic inputs and returns deterministic responses have limitations in terms of emulating the actual behavior of the engineered systems. On the other hand, the statistical model [4] receives statistical random inputs and returns statistical responses as shown in Figure 1. Thus, the statistical model is more appropriate to applying to the fault diagnosis of the engineered system with uncertainties than the deterministic model.

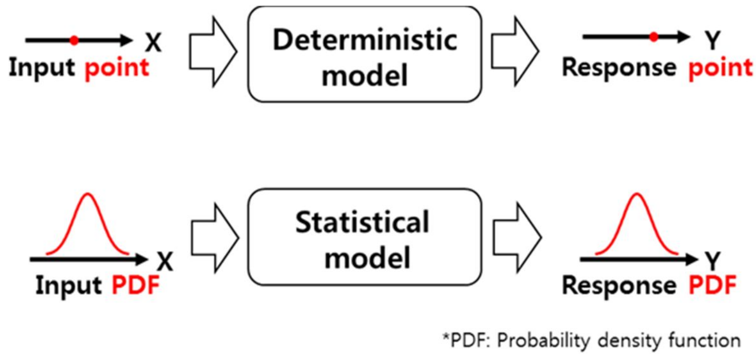


Figure 1 The difference between deterministic model and statistical model

For using the statistical model in engineered field, the statistical model should be validated. This process is called statistical model validation. [1-3] Statistical model validation means calibrating the computational model and testing validity of the calibrated model. In other words, statistical model validation includes statistical model calibration and validity check. [1-3] [5] The statistical model calibration is conducted by using calibration metric which adjusts the random input parameters of the computational model in order that the model response agrees with the experimental data. [5] Thus, the calibration metric has an important meaning in statistical model calibration. (The more details on statistical model validation is represented in Chapter 2.) However, some existing metrics for statistical model calibration including log-likelihood and Kullback-Leibler divergence (KLD) have several limitations in terms of calibration accuracy and efficiency. (The review on existing calibration metrics is described in Chapter 3.) So as to supplement these limitations, this study proposes a new metric for statistical model calibration, namely probability residual (PR). For testing the availability of PR to the real engineered system, the PR metric is applied to statistical model validation for a journal bearing rotor system.

1.2 Organization of Thesis

The ultimate goal of this study is to prove the high-performance of the probability residual (PR) metric in terms of the calibration accuracy and efficiency, and apply the PR metric to the statistical model validation of the actual journal bearing rotor system in normal state and abnormal state of rubbing. This paper is organized as follow. Chapter 2 reviews statistical model validation including model uncertainties, statistical model calibration and validity check. The review on fault diagnosis of a journal bearing rotor system is also covered in this chapter. Chapter 3 introduces a new calibration metric PR and its characteristics. For a comparative study, existing metrics for statistical model calibration such as log-likelihood, Kullback-Leibler divergence (KLD) are recommended. And then, the performance of the PR metric is evaluated with mathematical models including linear, nonlinear and elliptical examples. With each of the mathematical models, the statistical calibration is implemented to three kinds of the calibration metrics including log-likelihood, KLD and the PR metric in this chapter. In Chapter 4, case study, which employs the rotor-dynamics model with journal bearings in normal state and abnormal state of rubbing, is implemented. After the statistical model calibration with the PR metric, validity check with hypothesis testing based on area metric is conducted. Chapter 5 summarizes the contributions of this study, and the future works are discussed in this chapter.

Chapter 2. Literature Review

Chapter 2 represents literature review and the theoretical background for better understanding of this research. This chapter consists of two sections: statistical model validation including model uncertainties, statistical model calibration and validity check; fault diagnosis of a journal bearing rotor system. Some part of this chapter is supposed to be submitted to the journal of structural and multidisciplinary optimization (SMO).

2.1 Statistical Model Validation

Statistical model validation is originated from the research area of verification and validation (V&V). [1, 2] The model verification is defined as "the process of determining that a computational model accurately represents the underlying mathematical model and its solution" in American Society of Mechanical Engineers (AMSE). [2] And the model validation is defined as "the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model". [1, 2] [4] In brief, the model verification is focused on the relationship between computational model and mathematical model; the model validation is concentrated on the relationship between computational model and real world, e.g., experiment. The relationship among components of model V&V is shown in Figure 2. [6] [7] (The area of the verification is not a concern in this study. Because the computational model approximates solutions well as much as the mathematical model as the solver from various algorithms has been much developed recently.)

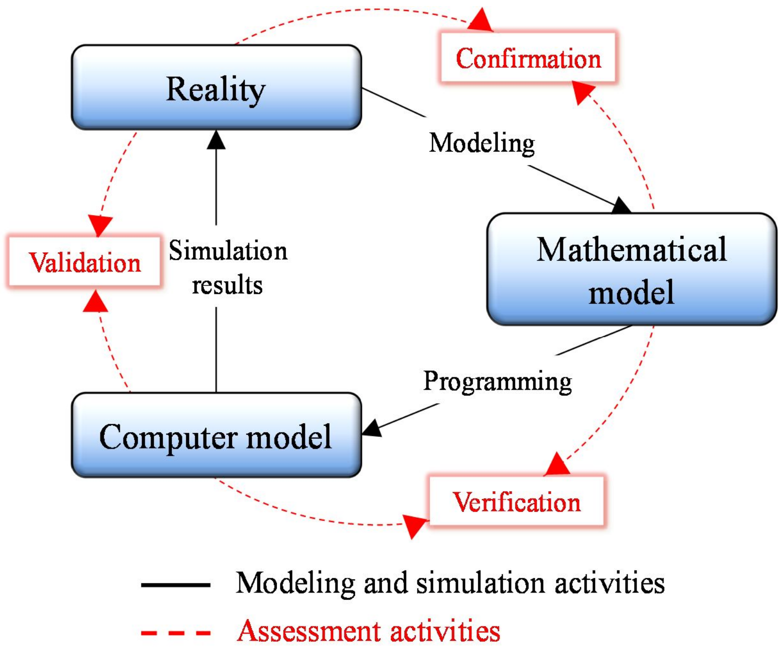


Figure 2 Simplified view of the model V&V process [4]

Figure 3 shows the procedure of model validation proposed by the ASME Standard Committee, in view of product manufacturing in the engineered system. [4] As shown in Figure 3, statistical model validation is composed of two parts: statistical model calibration and validity check. [1, 2] [4] [8] Since both the computational model and physical model have uncertainties, the results of them are represented as statistical distributions. Also uncertainties cause the disagreement between computational result (predicted result) and physical result (observed result). If then, the computational model should be adjusted to the physical model, which is referred to statistical model calibration. If the calibrated model is not valid, the model refinement is operated until satisfying validity check. More details on the model uncertainties, statistical model calibration and validity check are discussed in next sub-sections.

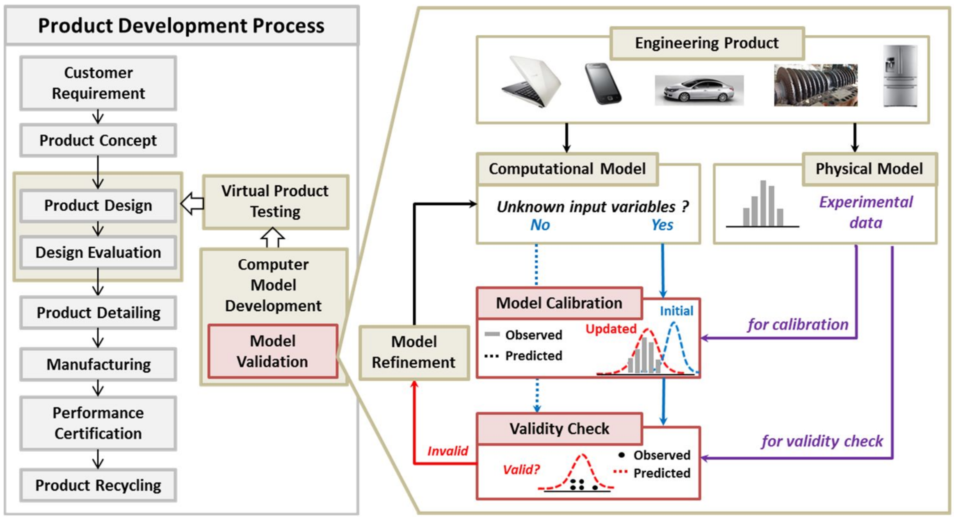
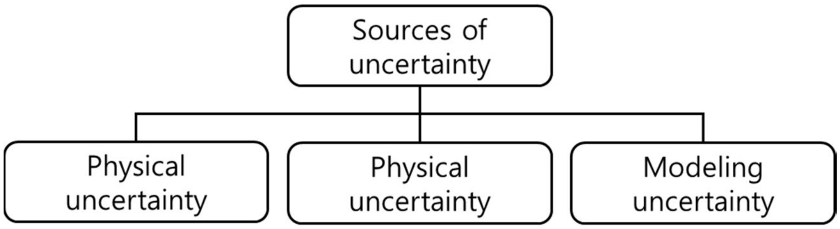


Figure 3 Model validation procedure [4]

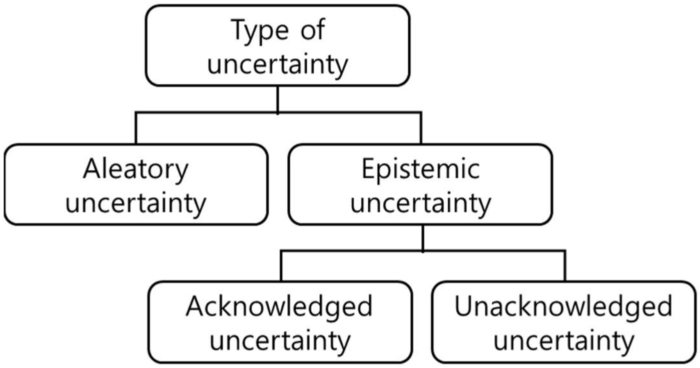
2.1.1 Model Uncertainties

Diverse uncertainties are included in engineered systems inherently. The sources of the model uncertainties are categorized into physical uncertainty, statistical uncertainty and modeling uncertainty as shown in Figure 4(a). [8] The physical uncertainty is inherent variation in physical quantity, and described by probability distribution. Material properties (tensile strength, Young's modulus, friction coefficient, stiffness coefficient, etc.), manufacturing tolerance (accuracy of milling machine, proficiency of craftsman, etc.), loading condition (force, torque, etc.), atmosphere condition (temperature, humidity, etc.) are contained in this kind of uncertainty. The statistical uncertainty is imprecise statistical estimation such as the type of the probability distribution, which depends on the sample size. Lack of data or improper sampling causes this kind of uncertainty. The modeling uncertainty originates from improper approximation such as inaccurate boundary conditions. Among these kinds of uncertainties including physical statistical and modeling uncertainty, due to the modeling uncertainty, the response of the computational model does not correspond to the experimental data, occasionally. Specifically, unknown random input parameters of the computational model can cause the modeling uncertainty, which results in discrepancy between computational result and experiment. In this case, the statistical model calibration, which adjusts the computational model to the experiment, is usually executed to the computational model.

The type of the model uncertainties is classified into aleatory uncertainty and epistemic uncertainty as depicted in Figure 4(b). The aleatory uncertainty, also called objective uncertainty, means irreducible uncertainty in spite of more relevant data. Because this sort of uncertainty resulted from inherent randomness of nature, it is impossible to reduce this kind of uncertainty. On the other hand, the epistemic uncertainty, also called subjective uncertainty, can be reduced with more relevant data. Epistemic uncertainty is divided into acknowledged and unacknowledged uncertainty. [9] The acknowledged uncertainty comes from assumptions ignoring several conditions for practical reasons on purpose of simplification or for just mistake. This uncertainty is reducible by regrading more complex conditions of engineered system. On the contrary, unacknowledged uncertainty is caused by lack of information or knowledge. This type of uncertainty cannot be easily identified.



(a)



(b)

Figure 4 Category of uncertainties, (a) Sources of uncertainty, and (b) Type of uncertainty

2.1.2 Statistical Model Calibration

The statistical model calibration includes variable screening, uncertainty characterization, uncertainty propagation, and optimization. [8] The variable screening is to determine the random input variables which make great impact on the computational response by sensitivity analysis. The uncertainty characterization is identifying the sort of the distribution of the random input variables considering physical characteristics of the engineered system. For example, in case of the journal bearing rotor system, the normal distribution is the most common distribution of bearing stiffness coefficient as the random input variable. The uncertainty propagation means transmission of the uncertainty from the random input variables to the response throughout the computational model. The sampling method such as Monte Carlo simulation (MCS); dimension reduction method including univariate dimension reduction (UDR) [10], bivariate DR (BDR) [11], and eigenvector DR (EDR) [12]; stochastic spectral method like polynomial chaos expansion (PCE) adaptive-sparse PCE [13] come under the category of the uncertainty propagation method. In process of the uncertainty propagation, the surrogate model can be adopted for efficiency on behalf of the computational model. The surrogate model, also called response surface, is simply composed of response grid calculated from the specific input points determined by design of experiment (DoE) method such as Latin hypercube sampling (LHS) [14]. The optimization [15, 16] is to optimize the statistical parameters of the unknown random input variables by minimizing the objective function. At this point, the calibration metric serves as the objective function in the optimization process. The calibration metric, the key factor for the optimization process in the statistical model calibration, quantifies the level of agreement or disagreement between probability density function (PDF) of the computational response and distribution of the experimental data.

In brief, the statistical model calibration is to infer the statistical parameters of the unknown random input variables where the computational response agrees with the experimental data by optimization process, and the calibration metric plays an important role in the optimization process as the objective function. When calibration metric has a minimum or maximum value, the iterative process of statistical model calibration is finished. At the same time the design variables, statistical parameters of random input variables, are determined as shown in Figure 5.

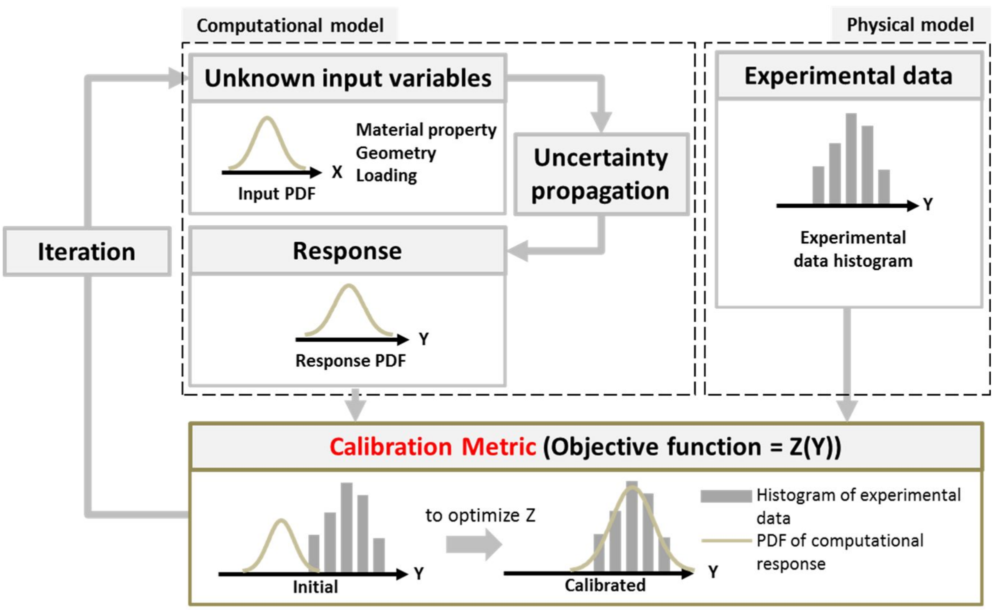


Figure 5 Process for statistical model calibration

2.1.3 Validity Check

The validity check is to confirm whether the calibrated model returns valid response with respect to other condition of experimental data. Specifically, the validity check means measuring the level of agreement between computational response and other condition of experiment, and representing the degree of validity quantitatively based on the hypothesis testing. This hypothesis testing enables us to make a decision whether we accept the calibrated model or not. [5] [17] [18] [19]

Among many kinds of the hypothesis testing methods such as the classical hypothesis testing [20] [21] and the Bayesian hypothesis testing [19] [22], this study focused on the classical hypothesis testing. The classical hypothesis testing includes statistical parameter-based methods (t-test statistic or F-test static [20]), full distribution-based methods (Anderson-Darling test, Cramer-von Mises test, Kolmogorov-Smirnov (K-S) test [21]), etc. The statistical parameter-based methods examine consistency of statistical parameters such as the mean and standard deviation of two distributions which is the observation (experimental data) and prediction (computational result). And, full distribution-based methods measure the differences between cumulative density functions (CDFs) of the observations and predictions. This classical hypothesis testing defines the null hypothesis (H_0) that the physical observations (experimental data) agrees with the prediction (computational result), and the alternative hypothesis (H_a) that the physical observations do not agrees with the prediction. [20] If the value of the test static representing the degree of agreement between two distributions quantitatively is located outside of the criteria region, the null hypothesis is rejected as illustrated Figure 6.

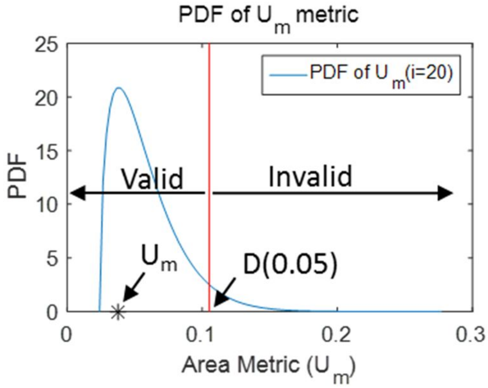
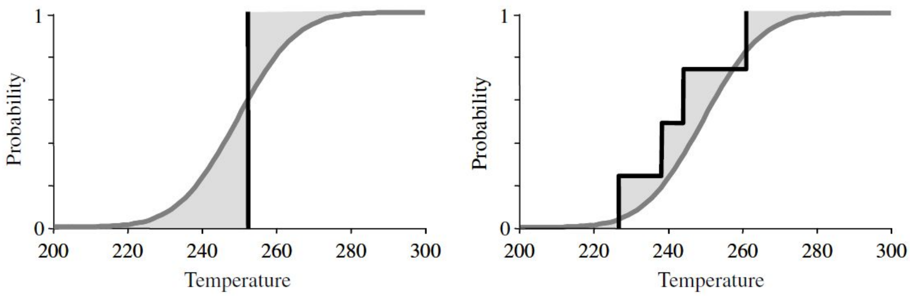


Figure 6 Hypothesis testing based on the area metric with 5% significance level

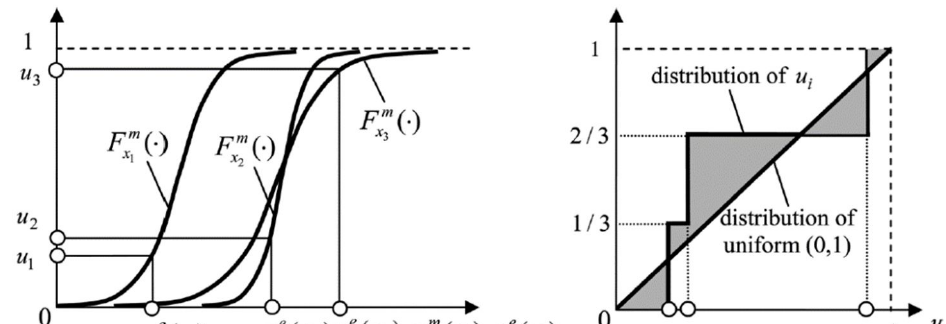
For the test static value of hypothesis testing, the area metric is commonly used. The area metric measures the area between CDF of the computational response and empirical CDF from the experiment. Figure 7 represents the concept of the area metric, where the smooth gray line means the computational response, black step line represents the empirical CDF of the experimental data, and the shaded area means the value of the area metric. [3] [23] [24]



(a) (b)

Figure 7 The concepts of the area metric, (a) data sets n=1, and (b) n=4 [25]

As shown in Figure 7, the area metric value in case of (a) is larger than that of (b) in consequence of epistemic uncertainty occurred by small size of data. Thus, for accurate validity check, the larger number of the experimental data is more advantageous than the smaller number of the experimental data. In case of few experimental data, u-pooling method [3] [23] [24] can be assistance to the area metric. With u-pooling method, all the experimental data at different validation sites can be pooled because any distribution can be changed to uniform distribution from zero to one when projected on its own CDF. Thus, u-pooling method leads the effect of increasing the number of data. In Figure 8(a), x_i, y^e, $F^m_{x_i}$, and u_i mean experimental point at different validation site, empirical PDF, marginal CDF of x_i and u values in universal probability scale, respectively. And how the area metric can be calculated by applying u-pooling method is illustrated in Figure 8(b).



(a) (b)

Figure 8 U-pooling method, (a) u-values at multiple validation sites; (b) area metric of mismatch between the empirical distribution of u-values and the standard uniform distribution [23]

2.2 Fault Diagnosis of a Journal Bearing Rotor System

Traditionally, the fault diagnosis of a journal bearing rotor system is executed based on vibration signals. [26] Most simply, when the amplitude of vibration signals obtained from sensor is increased than usual at the specific area, engineers notice the fault of the system and its position. However, only the amplitude of vibration cannot tell the details of the fault of a journal bearing rotor system such as rubbing, misalignment, oil whirl, etc. As the skills of the prognostics and health management (PHM) are developing, detail kinds of the fault can be classified based on health data. [27] The health data can be defined as numerical values representing the features of waveform on time domain or frequency domain such as the kurtosis, root mean square frequency (RMSF), etc. But, before determining the health data, characteristics of the system should be considered. For example, the crest factor which indicates how extreme the peaks are in a waveform on time domain can be the health data alarming the rubbing fault mode. Because the vibration signal at the rubbing fault mode comes to have many peaks than normal state. Actually, one fault mode is related to not one kind of health data but many kinds of health data, so that it is important to analyze the relation between many sorts of the heath data and fault mode of the system. So as to solve this issue, machine learning algorithms such as Fisher discriminant analysis (FDA), sparse vector machine (SVM), etc., can be assistance to classifying cluster of the health data. [28] As the artificial intelligence has been very interested recently, it is expected that the diverse deep learning algorithms will be able to contribute to the fault diagnosis. (Notify that this study will not focus on machine learning algorithms but health data as the response of computational model on behalf of experiment.)

Chapter 3. A New Calibration Metric – Probability Residual (PR)

Chapter 3 introduces a new calibration metric, probability residual (PR) after reviewing existing metrics for statistical model calibration. In this chapter, disadvantages of existing metrics and the sources of limitation of existing metrics are discussed. And the performance of the PR metric is evaluated in terms of calibration accuracy and efficiency. This chapter is supposed to be submitted to the journal of structural and multidisciplinary optimization (SMO).

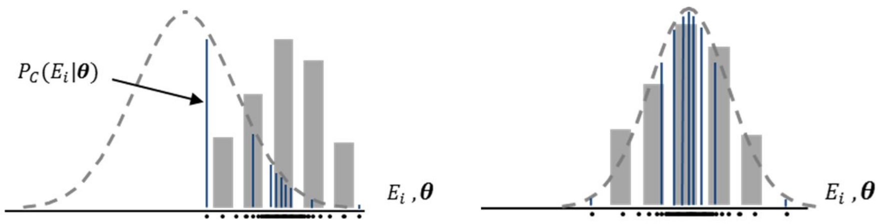
3.1 Review of Existing Calibration Metrics

The calibration metric, which quantifies the degree of similarity between PDF of the computational response and distribution of the experimental data, plays a key role in the statistical model calibration since the calibration metric serves as the objective function in the optimization process as denoted in Section 2.1.2. There are many existing metrics for calibration or validation such as relative error [29], root mean square error (RMS error) [30] [31], weight integrated factor (WIFac) [32] [33], etc. These metrics specialized in measuring the error of magnitude or phase. Geers and Thomas [34] suggested Geers metric in 1984, which combines individual metrics measuring each of response feature including magnitude and phase. For the statistical model calibration, however, statistical distributions rather than dynamic waves are treated as response. Also, the metric should be appropriate not to one determination but to recursive process and taken account of not only calibration accuracy but also calibration efficiency. For these reasons, there are few existing metrics suitable for statistical model calibration. In this section, log-likelihood and KLD are considered as the existing calibration metrics.

3.1.1 Log-likelihood

The Log-likelihood, one of the generally used metric in statistical fields, is defined as Eq. (1). [35] [36] E_i, $\boldsymbol{\theta}$ and P_C mean experimental data, statistical parameters vector, and PDF of the computational result respectively. P_C, represented by dashed line in Figure 9(a), is changed as the statistical parameters vector $\boldsymbol{\theta}$ varies. When the two distributions, PDF of the computational response and histogram of experimental data, are overlapped, the log-likelihood has a maximum value as shown in Figure 9(b). The reason of using the logarithmic operation to the likelihood can be summarized in two perspective. Firstly, as the mathematical operator, the logarithmic operation of the likelihood enables to convert the multiplication to an easy form of the summation, which improves the availability of the likelihood. Secondly, without the logarithmic operation, the multiplication result with the infinitesimal PDF value at the tail of the PDF makes likelihood always converge towards almost zero. It may cause convergence problems in optimization process.

$$\begin{aligned} Z_L &= \log_{10} \prod\nolimits_{i=1}^{n} P_C(E_i \mid \theta) \\ &= \sum_{i=1}^{n} \log_{10}(P_C(E_i \mid \theta)) \end{aligned} \tag{1}$$



(a) (b)

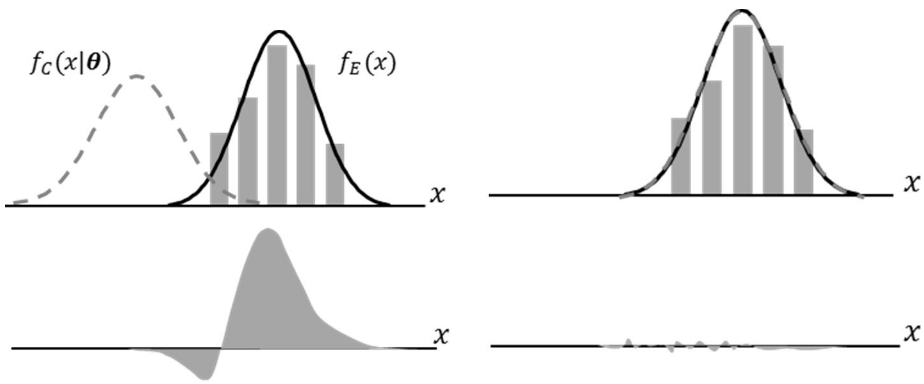
Figure 9 Concept of log-likelihood in condition of (a) initial and (b) optimized state

3.1.2 Kullback-Leibler Divergence (KLD)

The Kullback-Leibler divergence (KLD), also called relative entropy, is a widely used metric in information theory or probability theory as a means of class separability. [37] [38] Originally, KLD was used to quantify dissimilarity between two PDFs, but conversely, KLD also can measure the similarity between two PDFs. Eq. (2) represents the definition of KLD for two continuous PDFs. f_E and f_C in Figure 10(a) mean PDF of experiment and computational response respectively. Original KLD is basically an asymmetric measure as stated in Eq. (2). In other words, the value of $Z_{KLD}(f_E \parallel f_C)$ is not same with $Z_{KLD}(f_C \parallel f_E)$. This is a critical problem of KLD as a calibration metric since it recognizes the same degree of inaccuracy as a different value. Therefore, a symmetric form of KLD in Eq. (3) was devised, which enhances the availability of KLD by solving an above issue. [39] The KLD value is zero and the minimum when the two distributions are perfectly overlapped as illustrated in Figure 10(b).

$$Z_{KLD}(f_E \parallel f_C) = \int_{-\infty}^{\infty} f_E(x) \log_{10} \frac{f_E(x)}{f_C(x \mid \theta)} dx \tag{2}$$

$$Z_{KLD}(f_E, f_C) = Z_{KLD}(f_E \parallel f_C) + Z_{KLD}(f_C \parallel f_E) \tag{3}$$



(a) (b)

Figure 10 Concept of KLD in condition of (a) initial and (b) optimized state

3.1.3 Limitation of log-likelihood and Kullback-Leibler Divergence (KLD)

Although the logarithmic operation of the likelihood enables to convert the multiplication to an easy form of summation, it can also cause the poor calibration efficiency of which mechanism is described in Figure11. There are red and blue points of PDF values in Figure 11(a). If they are operated by logarithmic operator, they are put on the horizontal axis illustrated in Figure 11(b). (The color of the curve is separated based on decimal value of sensitivity.) In order to compare the sensitivity at the two points, the gradient method is selected. As a result, the gradient values at the red and blue points are almost no difference as shown in Figure 11(c). It implies that the values of the metric in the vicinity of the optimum point can be hard to be discriminated so that the optimization can be terminated at the inaccurate position in condition of the low tolerance. Since not only the log-likelihood but also Kullback-Leibler divergence (KLD) is expressed with a logarithmic term, KLD has the similar problem. This low sensitivity resulted from logarithm increases the number of the function evaluation or iterations in optimization process, which can cause poor efficiency problem. The detail results of sensitivity comparison will be dealt in Section 3.3.

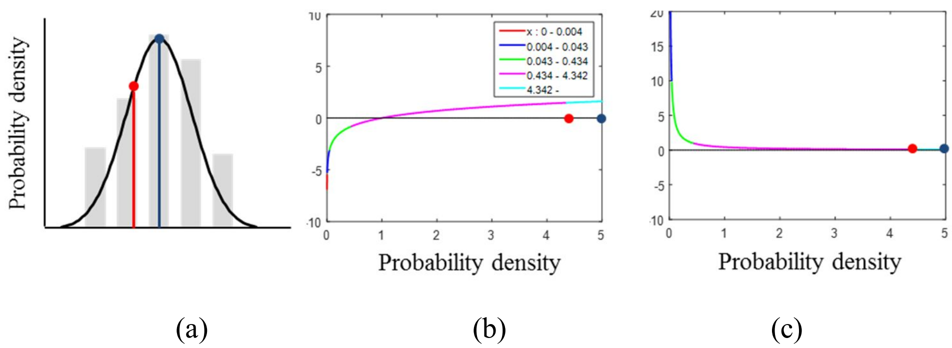


Figure 11 Low sensitivity problem of metrics with logarithm: (a) PDF, (b) log-operation plot, and (c) gradient plot

3.2 Proposed Calibration Metric – Probability Residual (PR)

In order to overcome previous limitation and to complement existing metrics, this study proposes a new calibration metric, namely probability residual (PR). The PR metric is defined as the sum of the product of the scale factor and the square of residuals as presented in Eq. (4). The scale factor, S as denoted in Eq. (4), is defined as Eq. (5) by Gauss' notation and arbitrary constant C. The scale factor converts the height of PDF to specific range, which enables to maintain the consistent sensitivity of calibration regardless of the distribution shapes. The squares of residual is the difference between the PDF of the computational response P_C and that of the experiment P_E. When the PDF of the computational model perfectly overlapped with experimental results, the PR value becomes zero like KLD as illustrated in Figure 12. This zero boundary owing to residual term notify whether optimization is correctly finished or not, which is one of the merit of the PR metric.

$$Z_{PR} = \sum_{i=1}^{n} S \times (P_E(E_i) - P_C(E_i \mid \theta))^2 \quad (4)$$

$$S = \frac{1}{C^{[\log_{10}(\max(P_E))]}} \quad (5)$$

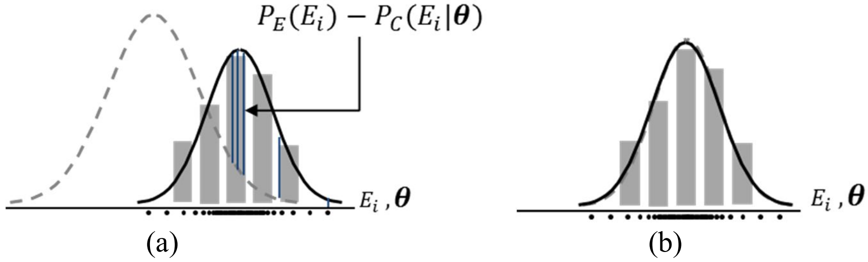
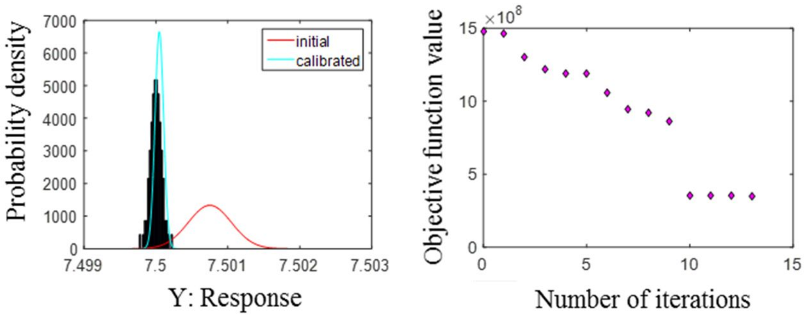


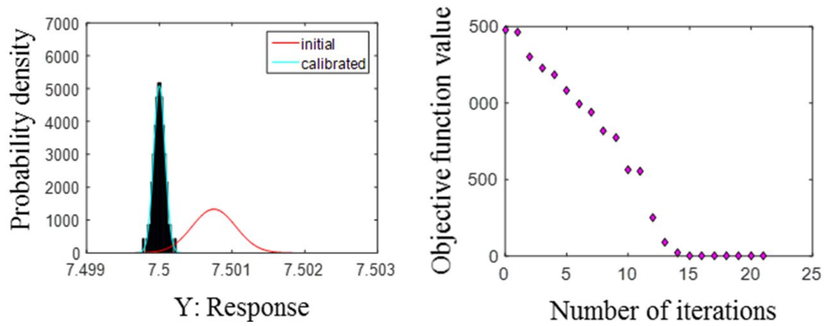
Figure 12 Concept of PR in condition of (a) initial (b) optimized state

3.2.1 Scale factor

The sensitivity of the calibration metric depends on the distribution shape of the experiment. Empirically, the wider the distribution shape comes to be, the more the number of function evaluations becomes in optimization process of the statistical model calibration. On the other hand, the number of function evaluations is decreased in case of the narrow distribution but the calibration accuracy can be worse in comparison with the case of the wide distribution. The feature of showing the consistent performance regardless of the distribution shapes is one of the critical conditions of a fine metric. In order to be a fine metric, PR is equipped with the scale factor which can solve this shape dependency problem of calibration metrics. The scale factor renders the PR metric maintain consistent calibration sensitivity as adjusting the PDF scale of the experiment and that of the computational result. Specifically, the scale factor defined as Eq. (5) is composed of the constant *C*, maximum value of experimental PDF (max(P_E)) and Gauss' notation ([ ]). Since the Gauss' notation in the scale factor includes logarithmic term, the value of the Gauss' notation functions as the indicators which transfer the degree of scaling effect to user instinctively. And, since the value of 'max(P_E)' is always fixed in statistical model calibration, the scale factor is controlled only by the arbitrary constant *C*. For example, if the value of *C* is defined as 100, the height of PDFs is determined to the range from 1 to 100 in any case of distribution shapes so that the wide distributions can be changed to be narrower than before and vice versa. Meanwhile, if the value of the ‘max(P_E)’ is come under between 1 and 10, the scale factor is always one regardless of the constant *C*. It means that there is no scaling effect. However, it means not the defunctionalization of scale factor but no necessity to applying the scale factor. Because the PDF range between 1 and 10 is a standard or desirable range in view of the optimization sensitivity.



(a)



(b)

Figure 13 Calibration result when the scale factor effect (a) off (C =1) and (b) on (C=100) in case of the narrow distribution

Table 1 Calibration result when the scale factor effect off (C=1) in case of the narrow distribution

	Y: Response Mean	Standard deviation	# of iterations	# of function evaluations
Initial state	7.5007	1.875e-4	0	0
Calibrated state	7.5	5.999e-5	13	56
MCS values	7.5	7.846e-5	none	none
%Error	6.32e-4	30.7784	none	none

	X: Random input variables Mean	Standard deviation
Initial state	10.001	2.5e-4
Calibrated state	10	8e-5
Solution	10	1.046e-4
%Error	6.31e-4	30.7768

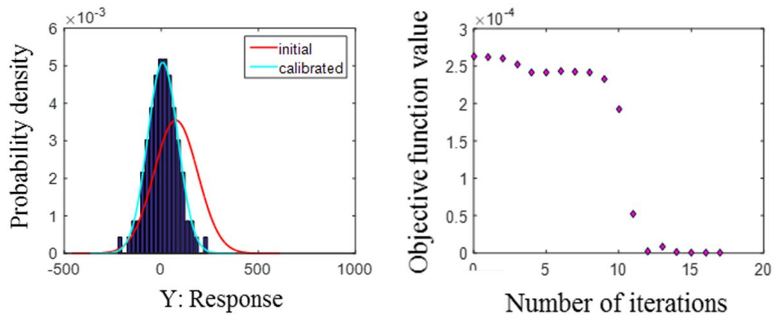
Table 2 Calibration result when the scale factor effect on (C=100) in case of the narrow distribution

	Y: Response Mean	Standard deviation	# of iterations	# of function evaluations
Initial state	7.5007	1.875e-4	0	0
Calibrated state	7.5	7.8465e-5	22	98
MCS values	7.5	7.846e-5	none	none
%Error	7.45e-7	5.52e-4	none	none

X: Random input variables

	Mean	Standard deviation
Initial state	10.001	2.5e-4
Calibrated state	10	1.046e-4
Solution	10	1.046e-4
%Error	7.24e-7	5.76e-4

Empirically, it is recommended that user determines an appropriate value of the constant *C* based on the range of transformed PDF from 1 to 10, or the objective function value from 0.1 to 1000. For testing the effect of the scale factor, the narrow and wide distributions are considered. In case of the narrow distribution, the statistical calibration is performed under 10^{-3} tolerance of x, as shown in Figure 13, Table 1, and Table 2. And, in case of the wide distribution, the statistical calibration is conducted under 10^{-6} tolerance of x, which is described in Figure 14, Table 3, and Table 4.



(a)

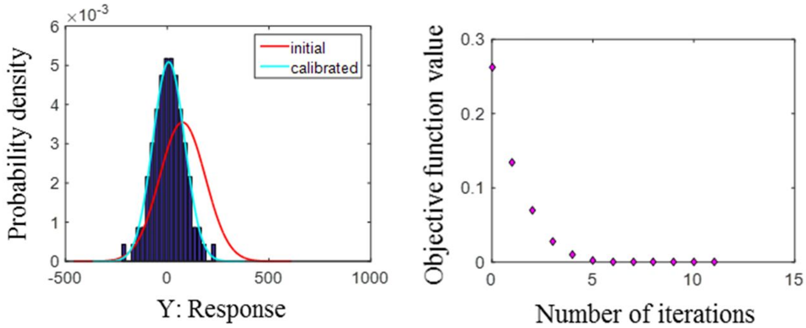


Figure 14 Calibration result when the scale factor effect (a) off (C =1) and (b) on (C=10) in case of the wide distribution

Table 3 Calibration result when the scale factor effect off (C=1) in case of the wide distribution

	Y: Response			
	Mean	Standard deviation	# of iterations	# of function evaluations
Initial state	75	150	0	0
Calibrated state	7.522	78.468	17	54
MCS values	7.5	78.466	none	none
%Error	0.2926	2.24e-3	none	none

	X: Random input variables	
	Mean	Standard deviation
Initial state	100	200
Calibrated state	10.029	104.6250
Solution	10	104.6214
%Error	0.2926	3.44e-3

Table 4 Calibration result when the scale factor effect on (C=10) in case of the wide distribution

	Y: Response			
	Mean	Standard deviation	# of iterations	# of function evaluations
Initial state	75	150	0	0
Calibrated state	7.5	78.466	11	37
MCS values	7.5	78.466	none	none
%Error	8.15e-6	1.59e-6	none	none

	X: Random input variables	
	Mean	Standard deviation
Initial state	100	200
Calibrated state	10	104.6227
Solution	10	104.6214
%Error	8.15e-6	1.21e-3

In these examples, all the responses (Y) are derived from the random input variables (X) which is projected on the response function of 'Y=0.75X'. Especially, the MCS values in Table 1, 2, 3 and 4 mean the responses obtained from the solutions in Table 1, 2, 3 and 4. So the '% error' at the table of response (Y) means the percentage error between calibrated state and MCS values.

In case of narrow distribution, when the scale factor effect on, the calibration accuracy is better than when the scale factor effect off as shown in Figure 13, Table 1 and Table 2. Although the number of iterations is increased from 13 to 22, the percentage error of the standard deviation of x is decreased from about 30% to 0%. Thus, it can be considered that the advantage from accuracy improvement is bigger than disadvantage from efficiency degradation in this case. In case of the wide distribution as represented in Figure 14, Table 3 and Table 4, the number of iterations is decreased from 17 to 11 and the percentage error of the mean of x is decreased from about 0.3% to 0. In this case, it is confirmed that both the calibration accuracy and the efficiency are improved by using the scale factor. These results indicate that the PR metric can exert an optimized performance as adjusting scale factor appropriately.

3.2.2 Square of residuals

As the quadratic form of the function, the term of square of residuals makes the PR metric a convex form at the vicinity of the operating point determined by statistical parameters vector, $\boldsymbol{\theta}$. It has highly significant meaning whether the objective function (cost function) is convex or not in view of optimization. Because the convex function has many strong points such as availability to satisfying Karush-Kuhn-Tucher (KKT) conditions or guaranteeing existence of the global optimum, etc. [40] The optimization problems are commonly concluded in finding the extremal values. If the performance function is a convex function, the local solution comes to be equal to global solution. [40] These characteristics of convex function have positive effects on the robustness and accuracy of optimization. Also, the square of the residuals term makes PR have a zero bound at the optimum point, which enables to notify whether the calibration is done correctly or not.

3.3 Performance Evaluation of the Calibration Metrics

The performance evaluation of the calibration metrics including log-likelihood, KLD and PR was conducted so as to compare the performance of PR with that of the existing calibration metrics. If the random input variables followed normal distribution, the normal, skewed, and bimodal response can be obtained by projecting the random inputs on the linear, nonlinear, and elliptical response functions, respectively as shown in Figure 15. These three response functions generate linear, skewed and bimodal response, which is common distribution of experiment. With each of the mathematical models, the three kinds of the calibration metrics including log-likelihood, KLD and PR were applied.

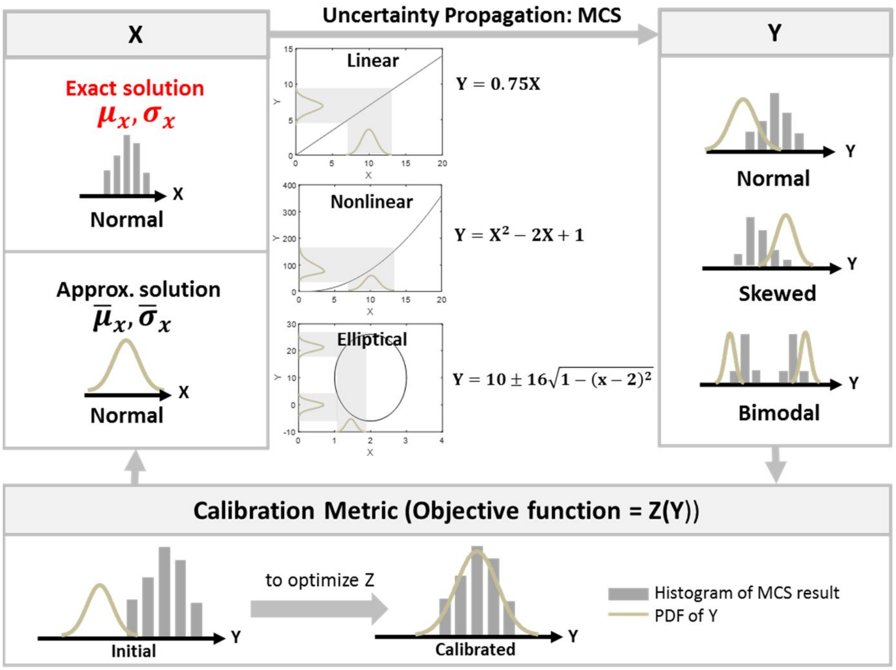


Figure 15 Process of the performance evaluation of PR

3.3.1 Comparative study of calibration metrics in terms of accuracy

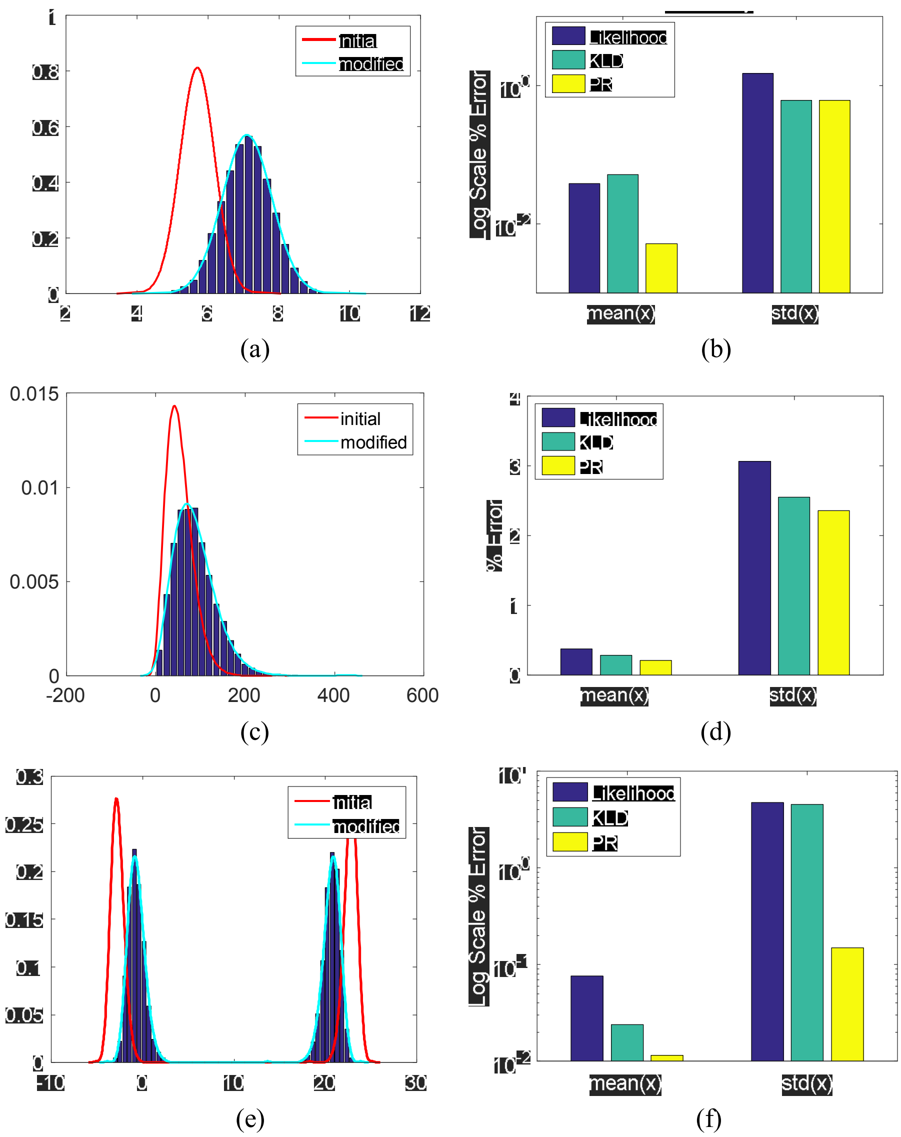


Figure 16 Calibration result of response by using PR in (a) linear, (c) nonlinear, and (e) elliptical case; percent error comparison in (b) linear, (d) nonlinear and (f) elliptical case

As shown in Figure 16(a), (c) and (e), the calibration is performed correctly. (It is denoted that the sample size used in these analytical examples is 10^6). As presented in Figure 16(b), (d) and (f), the PR metric performed better than other metrics including log-likelihood and KLD in terms of the calibration accuracy. The nonlinearity of the response function is increasing as progressing to the linear, nonlinear and elliptical case. The result represents that the PR metric shows better performance despite the condition of the highly nonlinear response functions owing to the convex form of square of the residuals.

3.3.2 Comparative study of calibration metrics in terms of accuracy

As illustrated in Figure 17, the PR metric shows the smallest number of function evaluations among above calibration metrics as changing the tolerance of x from 10^{-4} to 10^{-6}. (It is known that there is a tendency for the optimization accuracy to converge at the range from 10^{-4} to 10^{-6} tolerance and 10^6 sample size. [41]) This result implies that the PR metric shows better calibration efficiency than other metrics including log-likelihood and KLD because of the scale factor.

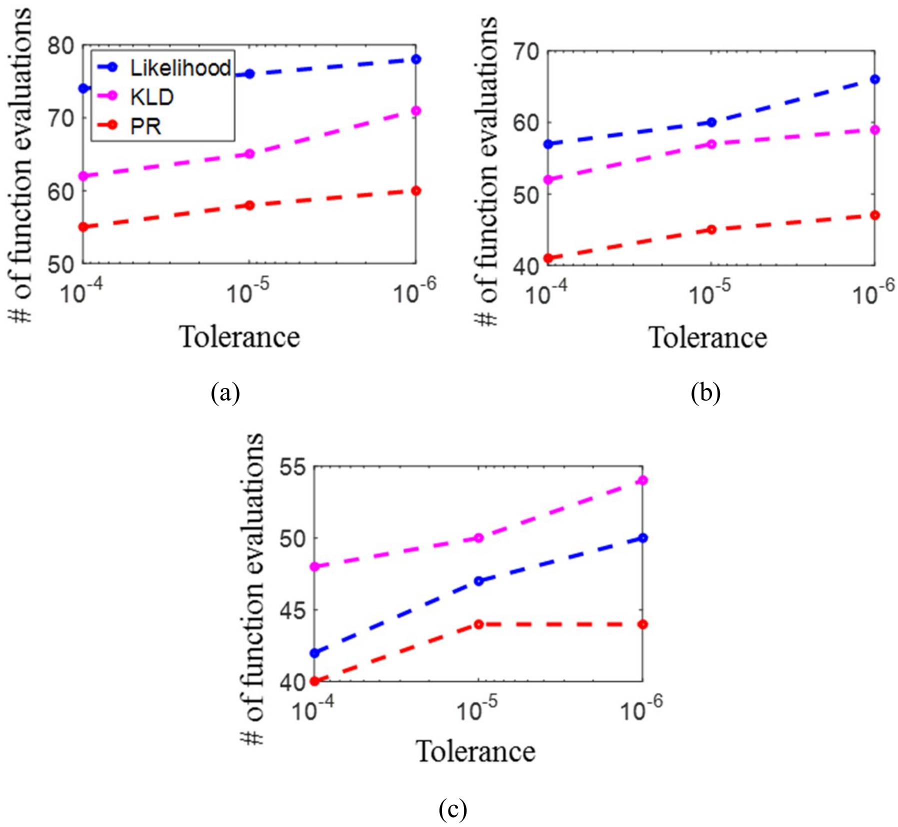


Figure 17 Number of function evaluations in (a) linear, (b) nonlinear, and (c) elliptical case

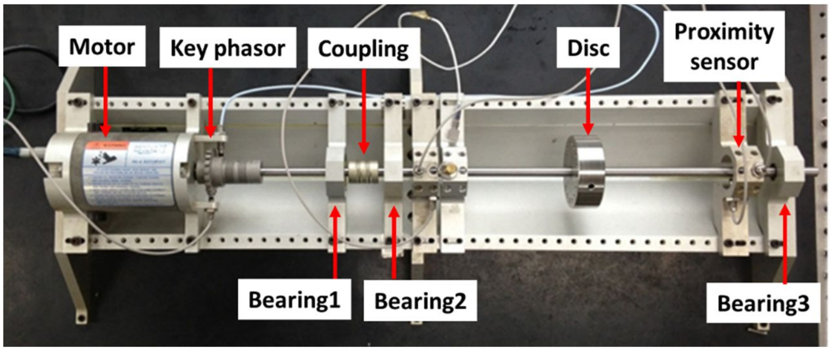
Chapter 4. Case Study: Statistical Model Validation of a Journal Bearing Rotor System

4.1 Hierarchical Framework for Statistical Model Validation

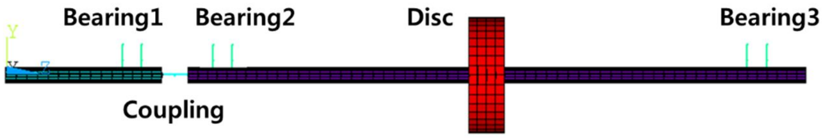
The statistical models of the journal bearing rotor system in normal and rubbing state is constructed and statistical model validation is conducted in this Chapter so as to diagnose the rubbing of the journal bearing rotor system. The first reason why the hierarchical framework [4] is chosen is availability to extend from the normal state to other states. The second reason is the efficiency and the accuracy of computational calculation. There can be many unknown random input parameters in single system. In this case, it would be better to calibrate the unknown parameters step by step hierarchically rather than calibrate them all at once. In this study, bearing stiffness and damping coefficient is calibrated at the normal state tier, and the normal contact stiffness is calibrated at the rubbing state tier. The details are discussed in chapter 4.1.2 and 4.1.3.

4.1.1 Description of a Computational Model

In this research, an RK4 journal bearing testbed was used to statistical modeling. This testbed was simply composed of two shaft and one disk. A computational model of this testbed was intended to obtain vibration signal so as to complement the scarcity of the experimental data in fault diagnosis. The details are represented in Figure18.



(a)



(b)

Figure 18 RK4 model: (a) RK-4 testbed, (b) Computational model of RK-4

4.1.2 Statistical Model Calibration in Normal State

In this tier of normal state, 1st critical speed is chosen as the response; bearing stiffness and damping coefficient is defined as the unknown random input variables from gradient based sensitivity analysis as a variable screening method. The Monte Carlo simulation (MCS) is adopted for the uncertainty propagation method; the PR metric is applied as a calibration metric as shown in Figure 19. For the efficiency, the surrogate model is used based on polynomial regression.

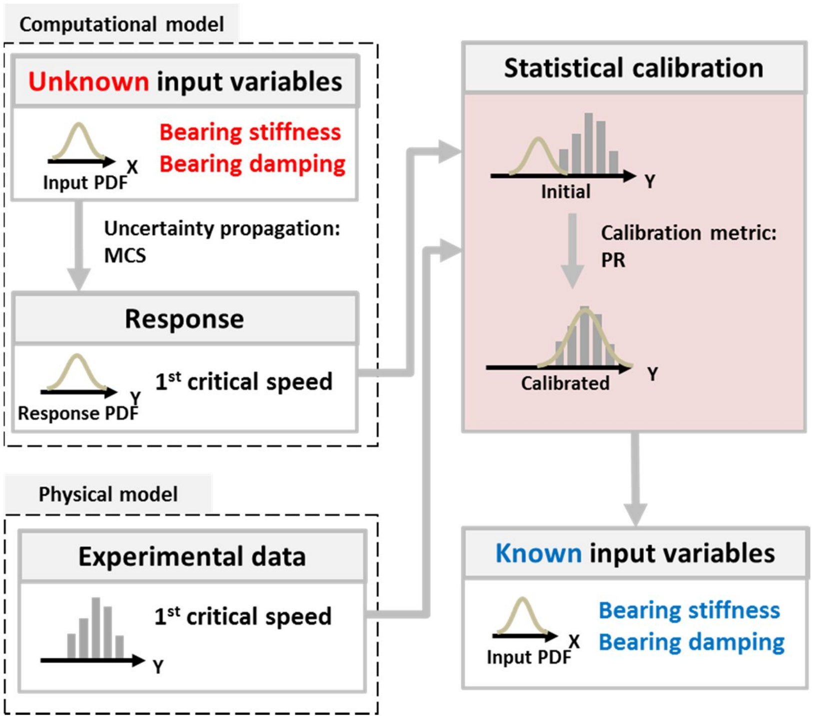
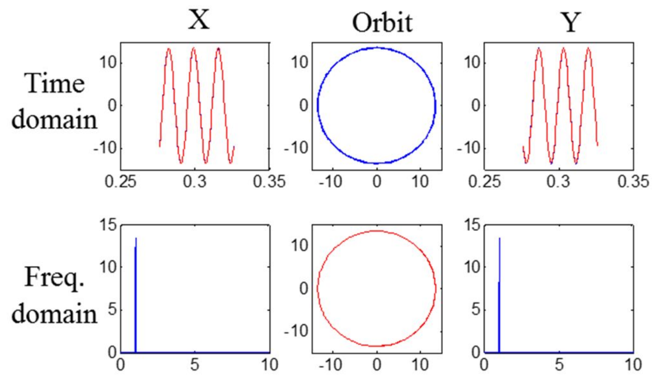


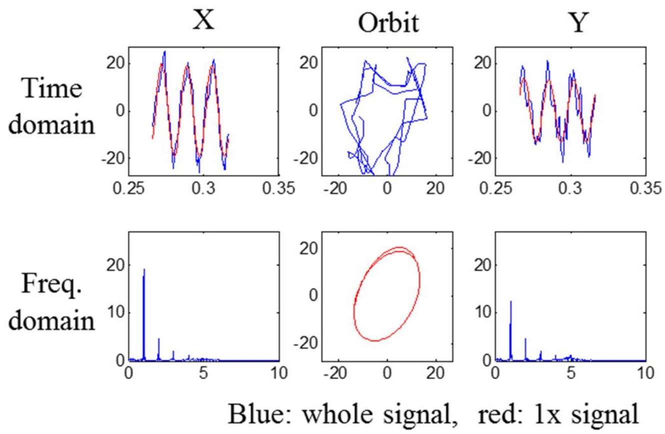
Figure 19 Process of the model calibration in normal state

4.1.3 Statistical Model Validation in Rubbing State

As referred in Section 2.2, the detail kinds of the fault can be classified based on health data. The health data can be defined as numerical values representing the features of waveform on time domain or frequency domain such as the kurtosis, root mean square frequency (RMSF), etc. But, before determining the health data, characteristics of the system should be considered. Figure 20 represents the characteristics of the normal state and rubbing state. Because the vibration signal at the rubbing fault mode has many peaks than normal state, the crest factor as denoted as Eq. (6) which indicates how extreme the peaks are in a waveform on time domain can be the health data alarming the rubbing fault mode. For this reason, in this tier of rubbing state, the crest factor is chosen as the response.



(a)



(b)

Figure 20 Vibration signal in case of (a) normal state and (b) rubbing state

$$H_{Crestfactor} = \frac{|x|_{peak}}{x_{rms}} \tag{6}$$

Considering the physical meaning with the rubbing in finite element analysis (FEA), the normal contact stiffness [42] [43] for penalty method [43] [44] was defined as the unknown random input variable since the computational model as shown in Figure 18(b) is for node to node contact. It is known that the penalty method is most proper to the condition of the node to node contact as represented in Table 5. (The information in Table 5 is cited from ANSYS APDL instruction manual.)

Table 5 Contact analysis method

	Penalty Method	Augmented Lagrange	Pure Lagrange	MPC*
Trait	Good convergence behavior (few iterations)	May require additional iterations if penetration is too large	May require additional iterations if penetration is too large	Good convergence behavior (few iterations)
Contact condition	Node - node	Node - surface	Surface - surface	Surface - surface
Geometry condition	Symmetric or asymmetric contact	Symmetric or Asymmetric contact	Symmetric or asymmetric contact	Asymmetric contact only
Contact behavior	Any type	Any type	Any type	Only bonded & no separation
Contact stiffness control	Normal contact stiffness	Normal contact stiffness	Tangential contact stiffness	Tangential contact stiffness

* MPC: Multi-point constraint

For the validity check, the experimental data is obtained by increasing unbalance level which imitates increasing the rubbing effect. The unbalance mass used for validity check is 0.9g. Finally, hypothesis testing based on area metric is conducted. The whole process represented in Figure 21.

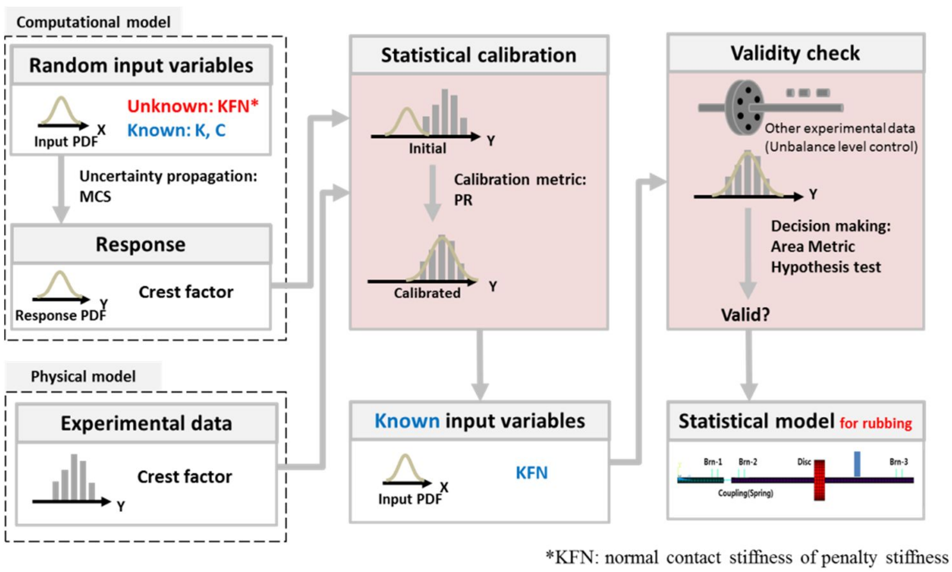


Figure 21 Process of the statistical model validation in rubbing state

4.2 Discussion

In the tier of normal state, the model calibration is performed. The calibration result is as shown in Figure 22, and Table 6. After model calibration, the statistical calibration and validation is conducted in the tier of rubbing state. The statistical calibration result is represented in Figure 23 and Table. 7, and the validation result is represented in Figure 24. With 5% significance level, the null hypothesis that the calibrated model for rubbing is valid could not be rejected. It indicates that the calibrated model by the PR metric is valid in view of engineering sense.

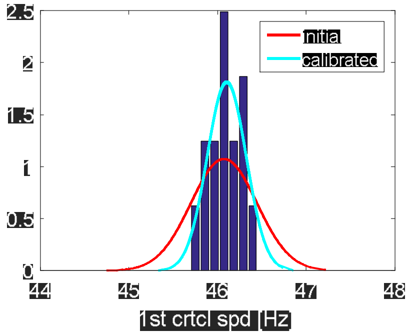


Figure 22 Calibration result with respect to 1st critical speed in normal state

Table 6 Calibrated statistical parameters of response in normal state

	1st critical speed	
	Mean	Standard deviation
Calibrated	45.6847	0.1908
Experimental	45.6935	0.1925
%Error	0.0193	0.8831

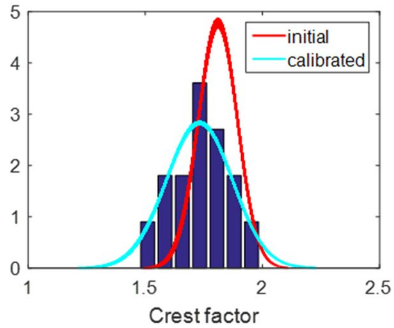
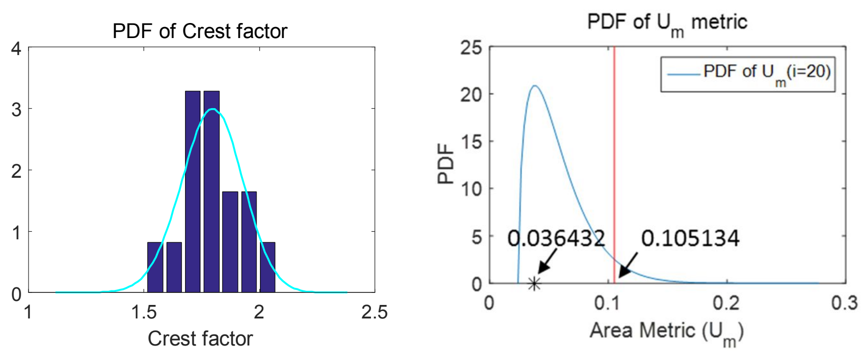


Figure 23 Calibration result with respect to crest factor in rubbing state

Table 7 Calibrated statistical parameters of response in rubbing state

	Crest factor	
	Mean	Standard deviation
Calibrated	1.7313	0.1400
Experimental	1.7313	0.1400
%Error	1.40e-4	4.59e-3



(a) (b)

Figure 24 Validity check result in rubbing state: (a) calibration result in rubbing state with respect to increased unbalance level, and (b) hypothesis testing

Chapter 5. Conclusions

5.1 Contributions

The statistical model calibration is to infer the statistical parameters of the unknown random input variables where the computational response corresponds to the experimental data by optimization process. Because the calibration metric serves as the objective function in optimization process, the statistical model calibration is significantly influenced by performance of the calibration metric. For the optimization, the calibration metric should be suitable not to one determination but to iterative process and taken account of not only accuracy but also efficiency. This study demonstrates the limitations of the existing metrics for statistical model calibration, i.e., log-likelihood, KLD, with regard to calibration accuracy and efficiency. In aspects of the calibration efficiency, the common cause of negative effect of existing metrics resulted from the logarithmic term. The logarithmic operation instigates low optimization sensitivity in the vicinity of the optimum point, and the low sensitivity triggers poor calibration efficiency by increasing the number of function evaluations and iterations.

To complement these problems, a new calibration metric, probability residual (PR), is proposed in this research. The merits of the proposed PR metric in comparison with other existing calibration metrics such as log-likelihood and KLD are summarized to three points. Firstly, the PR metric is a robust metric because it has a convex form of square of residuals term, which is the difference between PDFs of the computational response and experiment described in Chapter 3. It is significant whether the objective function is a convex form or not in view of optimization. Because the convex function has many strong points such as availability to satisfying Karush-Kuhn-Tucher (KKT) conditions or guaranteeing existence of the global optimum, etc. These characteristics of convex function have positive effects on the robustness and accuracy of optimization. Also, the square of the residuals term makes PR have a zero bound at the global optimum, which enables to notify whether the calibration is correct or not. Secondly, the PR metric is an efficient metric. The scale factor of PR defined in Chapter 3, controls value of the probability density function (PDF). Empirically, calibration fitting to the narrow and high-shape distribution can have characteristics of fine efficiency but low accuracy whereas calibration fitting to the wide and low-shape distribution can have features of poor efficiency but high accuracy. By adjusting proper value of the scale factor, the performance of the calibration metric, which depends on the shape of distribution, can be more enhanced. Thirdly, the PR metric shows stable performance regardless of the distributions of response functions including linear, nonlinear and elliptical case. On the other hand, the performances of the other metrics were fall as nonlinearity of the response is increasing. In other words, the PR metric is proven to be a promising metric to apply to not only linear system but also highly nonlinear system.

For testing the availability for PR to real engineered system, the hierarchical framework for statistical model validation of the journal bearing rotor system is performed by applying the PR metric in this study. After the bearing stiffness and damping coefficient are calibrated in the tier of the normal state, the normal contact stiffness is finally calibrated in the tier of the rubbing state. This hierarchical frame work for statistical validation of rotor dynamics model especially for the rubbing state is a pioneer work to the best of my knowledge. Through this case study, it is implied that not only the physical parameters but also artificial parameters only passable in FE model such as normal contact stiffness can be regarded as unknown random input variables. In the conclusion, the calibrated journal bearing rotor model in rubbing state is proven to be valid by the hypothesis testing where the null hypothesis that the calibrated model is valid could not be rejected with 5% significance level. It suggests that the proposed PR metric seems promising to be applied in building an accurate computational model.

5.2 Future Works

There can be unknown advantages of the existing metrics such as log-likelihood and KLD. Especially, the function of the logarithmic operation in calibration metric should be more studied. In order to improve the availability of the PR metric, the scale factor of PR needs to be generalized, and unknown limitations of the PR metric should be examined by testing with other diverse metrics besides log-likelihood and KLD. The further studies on existing metrics for statistical model calibration is required. The contour plot in the vicinity of the optimum point is recommended to prove the excellence of PR in terms of the accuracy in comparison with other calibration metrics. Also PR needs to be evaluated with various random inputs expressed to diverse statistical distributions as well as normal distribution. These issues are supposed to be covered and reflected in the next journal paper. Finally, it remains to be future works to apply PR to constructing the statistical models for other fault diagnosis including misalignment and oil whirl besides rubbing, or to other engineered systems above journal bearing rotor system.

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국문 초록

저널 베어링 회전체 시스템과 같은 공학 시스템의 유한요소 모델을 구축할 때에는, 통계적 모델이 불확실성을 내포하는 실제 공학 시스템의 거동을 잘 모사하기 때문에 주로 이용된다. 통계적 모델을 사용하기 위해서는 통계적 모델 검증을 거쳐야 하는데 통계적 모델 검증에는 통계적 모델 보정 과정이 포함된다. 모델 보정 과정에 필수적 요소 중 하나인 보정 척도는 해석 모델이 반환하는 결과와 실험데이터 간의 유사도 또는 비 유사도를 정량적으로 측정하는 역할을 한다. 그러나 Log−likelihood 와 KLD 같은 기존의 보정 척도는 로그항 때문에 필연적으로 발생하는 몇 가지 한계점을 안고 있기 때문에 정확한 유한요소 모델을 구축하는 데에 어려움이 있다. 이러한 한계점을 보완하기 위해 이 논문에서는 새로운 보정척도인 확률 잔차를 제안하였다. 확률 잔차는 척도보정상수와 잔차 제곱항의 곱의 합으로 정의가 된다. 척도보정상수는 확률밀도함수의 척도를 특정한 범위로 변환하여 보정의 효율성을 높이는 데 기여한다. 한편, 잔차 제곱항은 확률 잔차를 볼록 형태로 만들어 줌으로써 광대역 최적점의 존재를 보장해주는 데 기여한다. 확률 잔차의 성능을 검증하기 위해 이 논문에서는 수학적 모델과 정상상태, 마찰접촉 상태의 저널 베어링 회전체 모델을 이용하였다. 그 결과 확률 잔차의 성능이 기존의 Log−likelihood 나 KLD 에 비해 정확도와 효율성 측면에서 우수하였다. 또한 가설검정을 적용한 결과 확률 잔차로 보정한 저널베어링 로터 모델이 유효함을 확인하였다. 이로써 이 논문에서 제안된 확률 잔차는 정확한 유한요소 모델을 구축하는 데에 유도전망하게 확용될 수 있다.

주요어: 통계적 모델 검증
통계적 모델 보정
보정 척도
유효성 평가
저널 베어링 회전체 시스템
고장 진단
계층적 구조 계획법

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