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A generic probabilistic framework for structural health prognostics and uncertainty management

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ABSTRACT

Structural health prognostics can be broadly applied to various engineered artifacts in an engineered system. However, techniques and methodologies for health prognostics become application-specific. This study thus aims at formulating a generic framework of structural health prognostics, which is composed of four core elements: (i) a generic health index system with synthesized health index (SHI), (ii) a generic offline learning scheme using the sparse Bayes learning (SBL) technique, (iii) a generic online prediction scheme using the similarity-based interpolation (SBI), and (iv) an uncertainty propagation map for the prognostic uncertainty management. The SHI enables the use of heterogeneous sensory signals; the sparseness feature employing only a few neighboring kernel functions enables the real-time prediction of remaining useful lives (RULs) regardless of data size; the SBI predicts the RULs with the background health knowledge obtained under uncertain manufacturing and operation conditions; and the uncertainty propagation map enables the predicted RULs to be loaded with their statistical characteristics. The proposed generic framework of structural health prognostics is thus applicable to different engineered systems and its effectiveness is demonstrated with two cases studies.

1. Introduction

Activities of sensory health monitoring and life prediction are of great importance to critical decision-making processes such as maintenance replacement and component design in engineered systems. Research on real-time diagnosis and prognosis, which interprets data acquired by distributed sensor networks, utilizes these data streams in making critical decisions, and provides significant advancements across a wide range of applications. However, techniques and methodologies for health prognostics become application-specific. The objective of this study is to develop a generic probabilistic framework for structural health prognostics and uncertainty management, which is composed of four core elements: (i) a generic health index system with synthesized health index (SHI); (ii) a generic offline learning scheme using the sparse Bayes learning (SBL) technique; (iii) a generic online prediction scheme using the similarity-based interpolation (SBI); and (iv) an uncertainty propagation map for the prognostic uncertainty management.

Extracting health relevant information from heterogeneous sensory signals is the first important step for structural health prognostics. Different signal processing methods have been studied and employed to find out a set of the most

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important physical signals and construct system health indexes. The methods include regression and classification based methods [1,2], principle component analysis [3,4], time domain analysis, frequency domain analysis, wavelet analysis [5,6], and autoregressive moving average methods [7]. The health index which uses dominant physical signals as a direct health metric is referred to as the physics health index (PHI); an example is the impedance for battery health management. With the growing complexity of engineered systems and embedded sensor networks, the mapping of a multitude of heterogeneous sensory signals to the PHI is getting more and more difficult. To overcome the shortcoming of the PHI, we propose the synthesized health index (SHI), which is a normalized health index bounded in [0,1] through appropriate combination and transformation of multiple physical signals. The SHI facilitate a unified health characterization method for complex engineering systems with "1" representing healthy systems and "0" representing faulty systems. In the developed probabilistic framework, the health conditions of various engineered artifacts can be characterized by either the SHI or the PHI.

After extracting health relevant information from sensory signals and constructing system health indexes, system degradation characterization is another crucial task for structural health prognostics. Different machine learning techniques have been used for this purpose, such as support vector machine [8], artificial neural networks [9–12], Bayesian modeling [13], and Gaussian process regression [14,15]. For the prognostic technique to be real-time applicable, the efficiency is one of the key factors to be considered, as the data processing task for the prediction of the remaining useful lives (RULs) could become extremely time-consuming when a multitude of physical components and massive sensory data are involved. Besides the efficiency, the capability of handling uncertainties is another concern due to the uncertain nature of sensory signals in most engineering problems. The sparse Bayes learning scheme, for example the relevance vector machine (RVM) [16], is not only statistically loaded, but also has a great sparseness feature to employ only a few neighboring kernel functions. This sparseness feature of the background health knowledge will eventually speed up the online data processing and make possible a real-time RUL prediction, especially when sensory data are massive and heterogeneous for a set of physical components.

After the offline learning of the system degradation behavior, the remaining useful life can then be predicted by comparing the real-time sensory signals with the background health knowledge, with the help of appropriate life prediction techniques, such as artificial neural networks, neuro-fuzzy approach [17], Bayesian updating approaches [18], filtering techniques [19–21], and the approach based on the similarities [22]. One of the grand challenges in structural health prediction is managing various uncertainties in RUL prediction. The uncertainties mainly come from manufacturing variability over a population of physical artifacts, uncertain nature in operational conditions, and sensor noise. To properly manage the uncertainty, it is important to build statistically rich background health knowledge (or curves) and use an optimal combination of the health curves for accurate RUL prediction. The SBI predicts the RULs with statistically rich background health knowledge obtained under various manufacturing and operation conditions. The proposed generic framework of structural health prognostics is thus applicable to different engineered systems and its effectiveness is demonstrated with two cases studies.

The rest of the paper is organized as follows. Section 2 presents the proposed probabilistic structural health prognostics framework using a generic health index system, the SBL and SBI techniques, and the prognostic uncertainty management. Applications of the proposed methodology are presented in Section 3 and the conclusion of this work is given in Section 4.

2. A generic framework for structural health prognostics

This section is organized as follows. Subsection 2.1 overviews the generic structural health prognostics framework. Subsections 2.2 – 2.4 present three core elements in the generic health prognostics framework: the generic health index system, the generic offline learning scheme using the SBL technique, and the generic online prediction scheme using the SBI technique. Subsection 2.5 presents the uncertainty propagation analysis in the health prognostics process.

2.1. Generic framework for structural health prognostics

Fig. 1 outlines the proposed generic framework for structural health prognostics. The proposed framework is originated from the work in [22] and is generic in the sense that it offers a general approach to define structural health index, build background health knowledge, and predict RUL distributions.

The proposed health index system can model the health condition of an engineered component or system using two health index measures: (i) PHI and (ii) SHI. The PHI uses a dominant physical signal as a direct health measure. With the growing complexity of engineered systems and embedded sensor networks, the mapping of a multitude of heterogeneous sensory signals to a dominant health measure is getting more and more difficult. In such cases, we propose the SHI, which uses a normalized health index as a function of multiple physical signals.

The proposed offline learning process is of great importance to structural health prognostics because online prediction is made based on background health knowledge built in the offline learning process. In the offline process, it is very important to build statistically rich background health knowledge, which can account for manufacturing variability and operation uncertainty. On the other hand, the statistically rich health knowledge should be efficiently managed to enable real-time RUL prediction in the online prediction process, especially when sensory data are massive and heterogeneous.

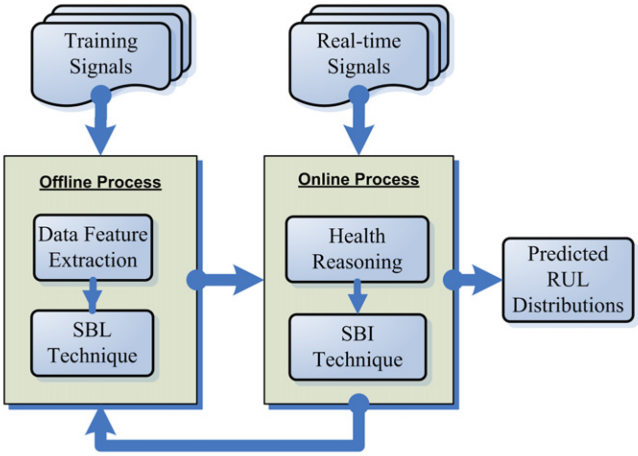

Fig. 1. A generic framework for structural health prognostics.

The SBL scheme, such as RVM, is a state-of-the-art technique for statistical regression that provides a regression result with not only a probabilistic form but also a great sparseness feature.

The online prediction process employs the background health information for the health prognostics using the SBI technique. This framework also enables the continuous update of the background health information through offline SBL and continuous update of the prognostics results in real-time with new sensory signals through SBI.

2.2. Generic health index system

This task considers massive training/testing sensory signals from embedded sensor networks over a complex engineered system. In this section, we propose a generic health index system composed of two distinguished health indexes: Physics Health Index (PHI) and Synthesized Health Index (SHI).

Physics Health Index (PHI): This health index system requires ample understanding of physics-of-failures of engineered component units. The PHI is thus applicable if sensory signals are directly related to physics-of-failures. In general, the PHI uses a dominant physical signal as a direct health metric. In the literature, most engineering practices of health prognostics are based on various PHIs. For example, the vibration signal has been used to characterize the health condition of the rolling element bearing by Gabreel et al. [13]; the radio frequency impedance has been used for the prognostics of electronic solder joint degradation [23]; the battery impedance value has been used to monitor the health condition of space application batteries [24]; and the capacitance of generator stator winding has been used for the wet bar detection and prognostics for water-cooled turbine generators [25], and so on. Just like the examples mentioned above, when sensory signals are directly related to physics-of-failures, it is straightforward and comprehensive to use the PHI for extracting health conditions of engineered system units. Otherwise, the application of the PHI is limited. It is expected that the mapping of a multitude of heterogeneous sensory signals to a dominant physical signal is getting more and more difficult with the growing complexity of engineering systems and embedded sensor networks.

Synthesized Health Index (SHI): The SHI is proposed as a possible solution to overcome the difficulty of the PHI described above. This health index is applicable when there is no dominant physical signal. One-dimensional SHI can be extracted from multi-dimensional sensory signals using advanced data processing techniques, such as weighted averaging methods [26], Mahalanobis distance measure [27], flux-based methods [28] and those methods [1–7] mentioned in the introduction section.

This study employs a linear data transformation method to construct the SHI, which has also been used in the work by Wang et al. [22]. Although this method is suitable for continuously collected sensory signals such as the data for case studies in this paper, other techniques and algorithms [26–28] can be used within the proposed generic prognostic framework to handle different type of signals such as discrete or binary sensory data. Suppose there are two groups of multi-dimensional sensory dataset that represent the system faulty and healthy conditions, Q_0 of $M_0 \times N$ matrix and Q_1 of $M_1 \times N$ matrix, respectively, where M_0 and M_1 are the numbers of dataset for system faulty and healthy conditions and N is the dimension of each dataset. With these two data matrices, an $N \times 1$ transformation matrix T can be obtained to transform the multi-dimensional sensory signal into the one-dimensional SHI as

$$T = (Q^T Q)^{-1} Q^T S_{off} \tag{1}$$

where $Q = [Q_0; Q_1]^T$, $S_{off} = [S_0, S_1]^T$, S_0 is a $1 \times M_0$ zero vector and S_1 is a $1 \times M_1$ unity vector. This transformation matrix T can transform any set of sensory signal at time t from the offline training, Q_{off}, or from online prediction process, Q_{on}, to the normalized SHI as $h(t) = Q_{off} \cdot T$ or $H = Q_{on} \cdot T$. The SHI can also be denoted as $h(t_i)$ for $i = 1, \ldots, N$, varying between 0 and 1. Since this SHI contains health condition signatures extracted from multi-dimensional sensory signals, it can be used to construct background health knowledge (e.g., predictive health degradation curves) in the offline training process and to further conduct the online prediction process.

The transformation of multi-dimensional physical signals into one-dimensional SHI is a process of information fusion, which enables a unified measure being used to characterize the system health condition. Thus, for the SHI to be useful for the health prognostics purpose, it should enable (1) the system degradation process being characterized, and (2) the similarity in the degradation process of different system units. In the prognostics framework, although the linear transformation is discussed here and will be employed for the case studies in this paper, different information fusion techniques can be used to construct the SHI. As the SHI will be constructed for all units in the same manner, and both the SHI and its uncertainty will be considered during the health prognostics process, the selection of SHI will affect the prognosis results by providing different levels of variations in the predicted RULs. Generally speaking, larger variations in SHI will lead to larger variations in the predicted RULs. In this sense, the way of constructing SHI that will lead to smaller variability between system units will result in better prognostics results.

2.3. Generic offline training scheme using SBL

The proposed offline training process aims at building background health knowledge using training sensory signals from offline system units. The SBL is employed to build the statistical form of background health knowledge, such as predictive health degradation curves for an engineered component of interest.

The SBL is a generalized linear model in a Bayesian form and it shares the same functional form of the SVM. The SVM is a pervasive machine learning technique using a linear combination of kernel functions centered at a subset of the training data, known as support vectors. Despite its widespread success, the SVM suffers from a critical limitation, being that it makes point predictions rather than statistical predictions. To overcome this problem, Tipping [16] has formulated this generalized linear model in a Bayesian form, named the RVM. It achieves comparable machine learning accuracy to the SVM but provides a full predictive distribution with substantially fewer kernel functions. To improve the efficiency and convergence of the RVM, several advances have been made for the original RVM, for example, the variational RVM [29], adaptive kernel RVM [30] and so on. This section briefly discusses a sparse linear regression model and the RVM with the sparse Bayesian learning for data regression and feature extraction.

2.3.1. A sparse Bayesian learning scheme

This paper proposes to use a sparse Bayes learning scheme using the RVM for the offline learning process. An unknown true health index function value $f(t)$ needs to be predicted at an arbitrary point t with a set of health index values, $h_1,\ldots,h_N$, measured at training points $t=\{t_1 ,\ldots, t_N\}$:

$$h(t)=f(t)+\varepsilon(t) \tag{2}$$

where $\varepsilon(t)$ is the measurement noise. The RVM is a special case of a sparse linear model:

$$h(t)=\sum_{i=1}^{N}\omega_i\phi(t,t_i)+\varepsilon(t) \tag{3}$$

where $\omega=\{\omega_1, \ldots, \omega_i,\ldots, \omega_N\}$ the weight vector and basis functions are formed by kernel functions $\phi(t, t_i)$ centered at the training points $t=\{t_1, \ldots, t_N\}$. The sparseness property enables the automatic selection of a proper kernel at each location by pruning all irrelevant kernels. A sparse weight prior distribution can be assigned, in such a way that a different variance parameter is assigned to each weight, as

$$p(\omega|\alpha)=\prod_{i=1}^{M}N(\omega_i|0,\alpha_i^{-1}) \tag{4}$$

where $\alpha=(\alpha_1,\ldots,\alpha_M)$ is a vector consisting of M hyper parameters, which are treated as independent random variables. To specify this hierarchical Bayesian inference model, prior distributions for α must be defined. For a scale hyper-parameter (α_i), it is common to use a Gamma prior distribution as

$$p(\alpha_i)=Gamma(a_i,b_i) \tag{5}$$

where a_i and b_i are the hyper-parameters and initially set to a flat Gamma distribution. The weight prior $p(\omega)$ can be obtained by integrating over the hyper-parameters as

$$p(\omega)=\int p(\omega|\alpha)p(\alpha)d\alpha \tag{6}$$

Assuming independent, zero-mean, Gaussian noise with a variance vector β^{-1}, i.e., $\varepsilon\sim N(0, \beta^{-1}I)$ where I is an identify matrix, we have the likelihood of the observed data as

$$p(h|\omega,\alpha,\beta)=N(h|\Phi\omega,\beta^{-1}I) \tag{7}$$

where Φ is either an $N\times N$ or an $N\times(N\times M)$ kernel matrix for the single and multi-kernel cases, respectively. This matrix is formed by all the basis functions evaluated at all the training points, i.e., $\Phi=[\phi(t_1),\cdots,\phi(t_N)]$ where $\phi(t_i)=[\phi(t_1-t_i),\ldots,\phi$

$(t_{i-1}-t_i)$, $\phi(t_{i+1}-t_i),\ldots,$ $\phi(t_N-t_i)]$. In order to make predictions using the Bayesian model, the parameter posterior distribution $p(\omega, \alpha, \beta|h)$ needs to be computed. However, this posterior distribution cannot be computed analytically owing to its complexity and thus approximations must be made through the decomposition of the posterior distribution and employing appropriate iterative optimization methods, such as marginal likelihood optimization [31], expectation maximization (EM) algorithms [32] or incremental optimization algorithms [33].

This SBL scheme can be applied on the system training health index dataset to construct the background health knowledge of system degradation with a set of predictive health degradation curves (h^p) with each represented in a statistical form as shown in Eq. (3). The following mathematical example illustrates how the SBL scheme can be applied on the health index data to construct the background health knowledge of system degradation. Suppose that a set of health index data h from $t=1$ (min) to $t=100$ (min) are obtained as

$$h=y+\varepsilon=\exp((1-t)/20)+0.075\sin((t-10)/2)+\varepsilon \tag{8}$$

where y is a function characterizing the pattern of health index data and ε is a zero mean Gaussian noise with standard deviation 0.05. The health index data as well as the function y are plotted in Fig. 2. By employing the Gaussian kernel function

$$\phi(x,x')=\exp\left(-\frac{(x-x')^2}{2\sigma^2}\right) \tag{9}$$

for RVM, the sparse Bayesian learning scheme obtained the predictive health degradation curve in a closed form as shown in Eq. (3). As shown in Fig. 2, with only 22 important relevance vectors (RVs), the SBL predictor achieved a good match with the exact health degradation curve represented by the function y. Most importantly, the predictive health degradation curve captured the uncertain nature of the health index data by using the statistics of the coefficients ω of the RVs. In this example, the mean vector and covariance matrix of the coefficients ω are partially shown in Table 1.

The SBL only employed a few critical basis points of the kernel functions to build the background health knowledge without losing the representativeness and uncertainty information. This desirable sparseness feature will speed up the online RUL prediction process and eventually enable the real-time RUL prediction. The SBL scheme provides an elegant approach to building the sparse linear models by treating the model parameters as random variables. With this treatment, both the statistical outputs and the desirable sparseness feature can be obtained. The SBL process can be carried out individually for different training unit which enables the background health knowledge to be built sequentially without

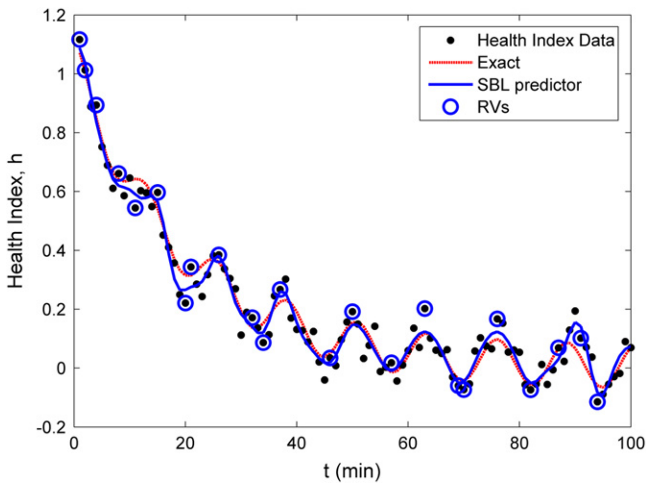

Fig. 2. Sparse Bayesian learning with health index data.

Table 1
Mean vector and covariance matrix of the ω.

Mean vector (1×24)	0.0781	1.4905	−0.8188	…	−0.2188
Covariance matrix (24×24)	1.72E−04	−2.30E−04	1.30E−04	…	−1.94E−04
	−2.30E−04	6.19E−02	−7.16E−02	…	2.59E−04
	1.30E−04	−7.16E−02	8.50E−02	…	−1.46E−04
	…	…	…	…	
	−1.94E−04	2.59E−04	−1.46E-04	…	9.53E−04

complicated retraining process and updated as more offline training units are gradually available. Applying the SBL to representing the SHI data enables the uncertainty quantification and management for RUL prognostics, which in turn leads to the prognostic results consisting of both an RUL estimate and its statistics. The accuracy of the SBL in representing the SHI data does affect the accuracy in RUL prediction. A more accurate model from SBL generally leads to more accurate RUL prediction but incurs a larger computational effort. The accuracy and the efficiency of the SBL have been intensively investigated in the literature and interested readers are directed to Ref. [16] for more information.

2.4. Generic online prediction scheme using SBI

The online prediction process aims at predicting the *RUL*s for online system units by employing a set of predictive health degradation curves built in the offline learning process. As the degradation curves are built probabilistically in the offline learning process, the uncertainty information will accordingly be propagated into the predicted RULs through the online prediction process. Following the similar procedure used in the deterministic RUL prediction by Wang et al. in [22], the online prediction process employed in the proposed probabilistic framework will also involve two steps: (i) determination of initial health condition and (ii) RUL prediction using the similarity-based interpolation (SBI).

2.4.1. Initial health condition

Component and system units tested in the online prediction process may have different initial health conditions, due to manufacturing variability or different service lives. So determination of initial health conditions for component units is of great importance to precise *RUL* prediction. In the first step, the health index data can be generated from testing sensory signals of online system units, based on either the PHI or the SHI. Then, the predictive degradation curve (h^p) as the background health knowledge will be employed to determine a time-scale initial health condition (T_0, or initial age) corresponding to the initial health condition where T_0 is a time state with the optimum fitting between the online health data and predictive health degradation curve. The optimum fitting can be formulated as

To determine T_0,

$$\min_{T_0} \sum_{j=1}^{N} (h(t_j) - h^p(t_j))^2, \text{ subject to } T_0 \in [0, L - \Delta t] \tag{10}$$

where $h(t_j)$ and $h^p(t_j)$ are the online health data and predictive health degradation data at t_j; N the number of data; T_0 the time-scale initial health condition (or initial age); Δt the time span ($= t_N - t_1$) of the online health index data; L the time span of a predictive health degradation curve, which is the total life of an offline system unit. This optimization process basically moves the online health index data $h(t_j)$ along the time axis to find the best time-scale initial health condition (T_0) by minimizing the fitting error with the predictive health degradation curve $h^p(t_j)$. As T_0 is determined by minimizing the fitting error as presented in Eq. (10), a non-monotonic degradation curve exhibiting a periodic pattern could lead to multiple solutions of T_0 from the curve fitting. As shown in Fig. 3, the degradation curves in Fig. 3(a) and (b) give unique solutions of the initial age T_0, whereas the curve in Fig. 3(c) gives two solutions of T_0. If the periodic pattern has been observed for the predictive degradation curve, a different technique for constructing the SHI should be sought. Once T_0 is determined, the projected remaining life of the online system unit on a given projected health degradation curve can be calculated as

$$RUL = L - \Delta t - T_0 \tag{11}$$

2.4.2. Similarity-based interpolation

This study proposes the similarity-based interpolation (SBI) to predict the *RUL* of an online unit. The predictive *RUL* of an online unit is a linear interpolation function in terms of the projected *RUL*s (L_i for $i = 1, \ldots, K$) of the offline units. The predictive *RUL* of an online unit can be expressed as

$$RUL = \frac{1}{W} \sum_{i=1}^{K} (W_i L_i) \quad \text{where} \quad W = \sum_{i=1}^{K} W_i \tag{12}$$

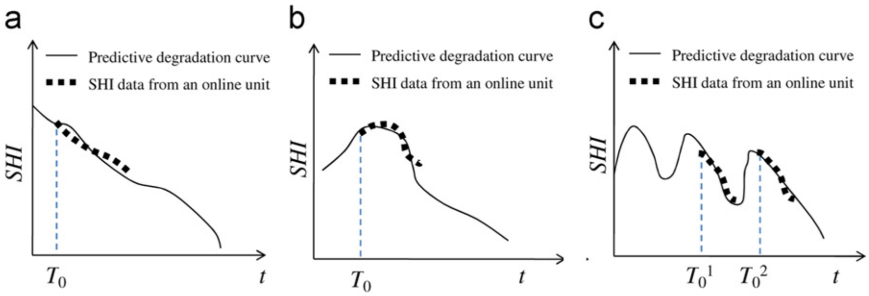

Fig. 3. Different characteristics of the predictive degradation curves and the resulting T_0.

where L_i is the projected *RUL* on the *i*th predictive health degradation curve and W_i the *i*th similarity weight. Then the predictive *RUL* of an online unit will be primarily determined as a linear function of L_i having larger degrees of similarity. Following the work by Wang et al. [22], the similarity weights can be defined through the sum-squared error as

$$W_i = \left[\sum_{j=1}^{N} (h_i(t_j) - h_i^p(t_j))^2\right]^{-1} \tag{13}$$

It is obvious that a larger weight is given to an offline unit with a greater similarity to the online unit. In other words, the offline units with larger weights have a greater similarity to the online unit in terms of manufacturing and service conditions. The similarity weights must be random due to the randomness in the predictive health degradation curve, $h_i^p(t_j)$, in Eq. (3). Using the random samples of $h_i^p(t_j)$, we can generate random sample sets of both similarity weights and projected *RUL*s (L_i for $i=1,\ldots,K$) and then construct the histogram of the predictive *RUL* of an online unit using Eq. (12).

The online prediction scheme with SBI for a given online unit of interest can be carried out repeatedly with gradually available sensory signals, and the prognostic results can be updated in real time. In the current study where the similarity-based interpolation is used for RUL prediction, the distinction between a SHI and PHI does not make a difference in the procedures for RUL prediction. However, the PHI provides the flexibility to incorporate a variety of the RUL prediction methods into the prognostics framework so as to make this framework generic.

2.5. Prognostic uncertainty management

Uncertainty management is of great importance for the health prognostics as it provides decision makers with the statistical information of predicted RULs. In the proposed prognostics framework, the uncertainty management conducted in both the offline learning and online prediction processes propagates the uncertainty in sensory signals to the statistical information of predicted RULs.

During the offline learning process, the SBL scheme using the RVM builds the background health knowledge by constructing the predictive degradation curves using the health index data. As the predictive health degradation curve in Eq. (3) is statistically obtained, the uncertainty information contained in sensory signals can be characterized with the statistics of the coefficients of the important RVs. Random samples of these coefficients can be generated based on the obtained statistics from SBL. Building Eq. (3) repeatedly with these random samples of the coefficients ω leads to a set of random realizations of the predictive degradation curves. In the aforementioned mathematical example, the uncertainty management in the offline learning process can be conducted accordingly, as Fig. 4 shows the random samples of the coefficients ω generated based on their statistics in Table 1 and Fig. 5 shows a set of random realizations of the predictive degradation curves.

With these random realizations of the predictive health degradation curve in Eq. (3) the uncertainty management in the online RUL prediction process can analyze the uncertainty propagation of raw sensory signals to the projected RULs. As discussed in subsection 2.4, the RUL for an online unit can be predicted using the SBI based on the mean predictive degradation curves in the background health knowledge. It is noted that different mean predictive health degradation curves are generated based on different offline units in the offline training process, whereas each mean predictive health

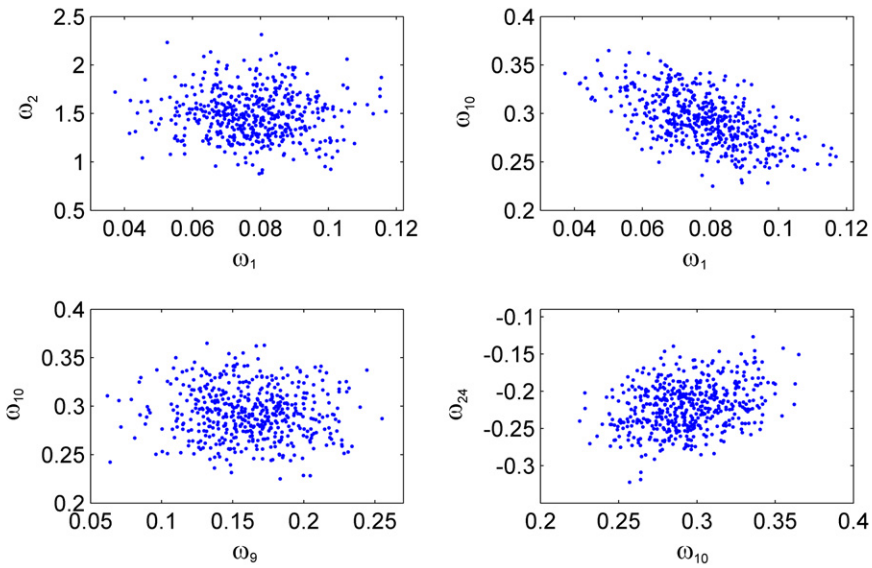

Fig. 4. Random samples for the coefficients in SBL.

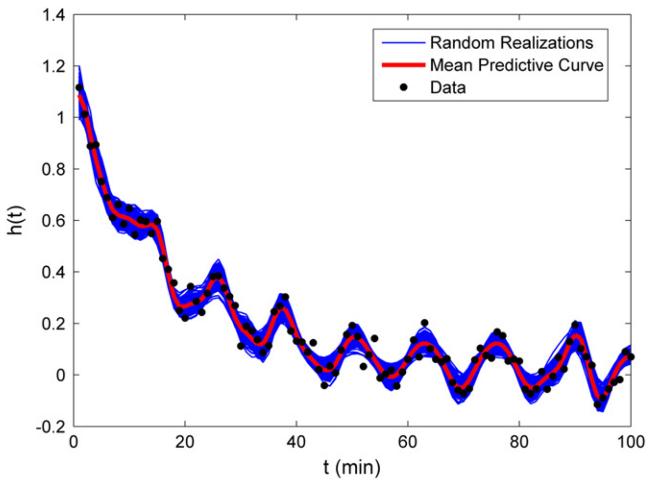

Fig. 5. Random realizations for the predictive degradation curve.

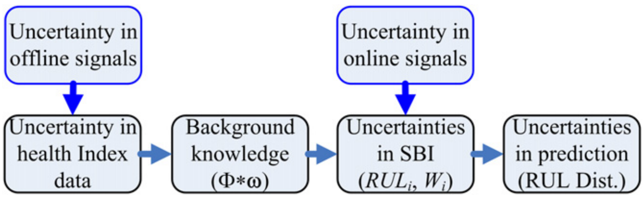

Fig. 6. Uncertainty propagation map in the structural health prognostics framework.

degradation curve has its own random realizations as shown in Fig. 5. To account for the uncertainties brought in by sensory signals, the RUL prediction process that matches the online degradation curves with the offline training curves, as discussed in subsection 2.4, is repeated for each set of the random realizations of the offline degradation curves. For example, with the i^{th} set of the random realizations of all offline units, the i^{th} predicted remaining useful life, RUL_n, for an online unit can be obtained based on Eq. (12) as

$$RUL_n = \frac{1}{W_n}\sum_{i=1}^{K}(W_{n,i}L_{n,i}) \quad \text{where} \quad W_n = \sum_{i=1}^{K} W_{n,i} \tag{14}$$

where $W_{n,i}$ is the similarity weight for the n^{th} random realization of the i^{th} offline unit and $L_{n,i}$ is the predicted RUL of the online unit based on n^{th} random realization of the i^{th} offline unit. Repeating the online RUL prediction process with a total number of N ($n=1, 2, \ldots, N$) sets of random realizations of all predictive degradation curves gives N different RUL values ($RUL_1, RUL_2, \ldots, RUL_N$) for the online unit. Finally, statistical information of the predicted RUL can be extracted from these N different RUL values. As more random curve realizations of offline units result in higher computational effort for the RUL prediction process, the authors suggest that $500-1000$ numbers of random realizations for each offline unit should be used to achieve a good balance between the speed of RUL prediction and the quality of RUL distribution.

The uncertainty propagation map for structural health prognostics is shown in Fig. 6. In the offline training process, uncertainties in the raw data are propagated to the health index data. The SBL technique then employs the uncertainties of the health index data to build the predictive health degradation curves in a stochastic fashion. Finally, in the online prediction process, the SBI predicts the RUL of an online system unit in a stochastic form, as shown in Eq. (12), using the predictive health degradation curves of all offline system units.

3. Case studies

This section demonstrates the effectiveness of the proposed generic framework for structural health prognostics with two case studies: (i) IEEE Prognostics and Health Management (PHM) 08 Challenge problem and (ii) electric cooling fan.

3.1. IEEE PHM 08 Challenge problem

The dataset provided by the 2008 IEEE PHM Challenge problem consists of multivariate time series signals that are collected from an engine dynamic simulation process. Each time series signal comes from a different degradation instance of the dynamic simulation of the same engine system [34]. The data for each cycle of each engine unit include the unit ID, cycle index, 3 values for an operational setting and 21 values for 21 sensor measurements. The sensor data were contaminated with measurement noise and each engine unit starts with different initial health conditions and manufacturing variation which is unknown. It is found that three operational settings have a substantial effect on engine degradation behaviors and result in six different operation regimes as shown in Table 2. The 21 sensory signals, as detailed in Table 3, were obtained from six different operation regimes [34]. The dataset was divided into training and testing subsets. The sensory signals were obtained from 218 offline engine units, so the number of training dataset is 4578 ($=21\times 218$) in total. The unit operated normally at the start of each time series and stopped until a fault condition was developed. The fault grows in magnitude until system failure, at which time, one or more limits for safe operation have been reached. There is no specific failure threshold defined. In the testing dataset, the time series signal ends some time prior to system failure. The objective of the problem is to predict the number of remaining operational cycles before system failure in the testing dataset.

3.1.1. Adjusting cycle index

To account for different initial health conditions, an adjusted cycle index is proposed as $C_{adj}=C–C_f$ where C is the operational cycle of a training engine unit and C_f the cycle-to-failure of the training engine unit. The cycle index 0 indicates an engine unit failure whereas a negative cycle index corresponds to an operational cycle prior to the failure. By setting the unit failure as a baseline, we can clearly display and conveniently use the health degradation behaviors of different offline training units with different initial health conditions and degradation paths.

3.1.2. Sensor signal screening

Among 21 sensory signals, some signals contain no or little degradation information of an engine unit whereas the others do. To improve the *RUL* prediction accuracy and efficiency, important sensory signals must be carefully selected to characterize degradation behavior for engine unit health prognostics. This study thus intended to screen sensory signals by observing the degradation behaviors of the 21 sensory signals. Among these 21 sensory signals detailed in Table 3, seven of them (2, 3, 4, 7, 11, 12, and 15) were selected in this study. Readers are suggested to find the detailed information regarding the sensor signal screening from Ref. [22].

Table 2
Six different operation regimes.

Regime ID	Operating parameter 1	Operating parameter 2	Operating parameter 3
1	0	0	100
2	20	0.25	20
3	20	0.7	0
4	25	0.62	80
5	35	0.84	60
6	42	0.84	40

Table 3
Description of the sensor data for the challenge problem.

Index	Symbol	Description	Units
1	T2	Total temperature at fan inlet	°R
2	T24	Total temperature at low pressure compressor(LPC) outlet	°R
3	T30	Total temperature at high pressure compressor (HPC) outlet	°R
4	T50	Total temperature at low pressure turbine (LPT) outlet	°R
5	P2	Pressure at fan inlet	psia
6	P15	Total pressure in bypass-duct	psia
7	P30	Total pressure at HPC outlet	psia
8	Nf	Physical fan speed	rpm
9	Nc	Physical core speed	rpm
10	Epr	Engine Pressure ratio	–
11	Ps30	Static pressure at HPC outlet	psia
12	Phi	Ratio of fuel flow to Ps30	pps/psi
13	NRf	Corrected fan speed	rpm
14	NRc	Corrected core speed	rpm
15	BPR	Bypass ratio	–
16	farB	Burner fuel – air ratio	–
17	htBleed	Bleed enthalpy	–
18	Nf_dmd	Demanded fan speed	rpm
19	PCNfR_dmd	Demanded corrected fan speed	rpm
20	W31	HPT coolant bleed	lbm/s
21	W32	LPT coolant bleed	lbm/s

3.1.3. Building SHI

As discussed above, seven sensory signals were used for engine prognostics study. Based on the signals, we built the SHI to represent the engine health degradation process. Different transformation matrices T_k must be constructed using Eq. (1) for six different operation regimes ($k=1-6$) because health degradation paths strongly depend on operation conditions. In doing so, different Q_0 and Q_1 matrices must be built for different operation regimes. For a given operation regime, the health index data to represent system failure and healthy conditions must be carefully identified to build Q_0 and Q_1. In this study, Q_0 was created with the sensory data in a system failure condition in which the adjusted cycle index is between -4 and 0. Similarly, Q_1 was constructed with the sensory data in a healthy condition in which the adjusted cycle index is smaller than -300. Different Q_0 and Q_1 can be created by repeating this process for all six different operating regimes. Table 4 presents the constructed 7×6 transformation matrix T_k where each column is a transformation vector for the corresponding operation regime. The dots in Fig. 7 represent the SHI data obtained using $H=T_kS_{off}$ with the training dataset of an offline engine unit.

3.1.4. Sparse Bayesian learning on SHI

Fig. 8 displays the randomly realized SHI data and the randomness is mainly due to the measurement noise from the signals. The RVM regression can be used to model the SHI data in a stochastic manner. As discussed in subsection 2.3, the RVM is Bayesian representation of a generalized sparse linear model, which shares the same functional form with the SVM. In this study, the Gaussian kernel function shown in Eq. (9) was used as a basis function for the RVM. To build the predictive health degradation curves ($h_i^p(t)$, $i=1,\cdots,218$) for 218 offline engine units, the RVM regression model can be formed with statistical coefficient vector (ω) in the generalized sparse linear model of Eq. (3).

Fig. 7 shows the health degradation curve with a desirable sparseness by only employing a small set of critical data points. Besides, the regression model gives both the mean and the variation of the predictive health degradation curve, as shown in Fig. 8. These predictive health degradation curves for the offline units altogether construct the background health knowledge which characterizes the system degradation behavior. Later, this background health knowledge can be used for modeling the predictive *RUL* distributions of online engine units. Some degradation curves for this challenge problem are exemplified in Fig. 9.

Table 4
Transformation matrix (T) for the SHI.

Regime 1	Regime 2	Regime 3	Regime 4	Regime 5	Regime 6
−0.03352	0.00420	0.01725	0.07551	0.04861	0.06308
−0.00358	−0.00571	−0.01046	−0.00551	−0.00720	−0.01003
−0.00760	−0.00741	−0.00624	−0.00695	−0.00891	−0.01105
0.03902	0.06381	0.05371	0.04381	0.05489	0.03470
−0.29961	−0.34434	−0.30928	−0.39681	−0.51199	−0.50965
0.07080	0.05048	0.07701	0.06448	0.08791	0.10163
−0.67360	−1.36813	−1.62036	−2.68974	−1.25800	−0.49316

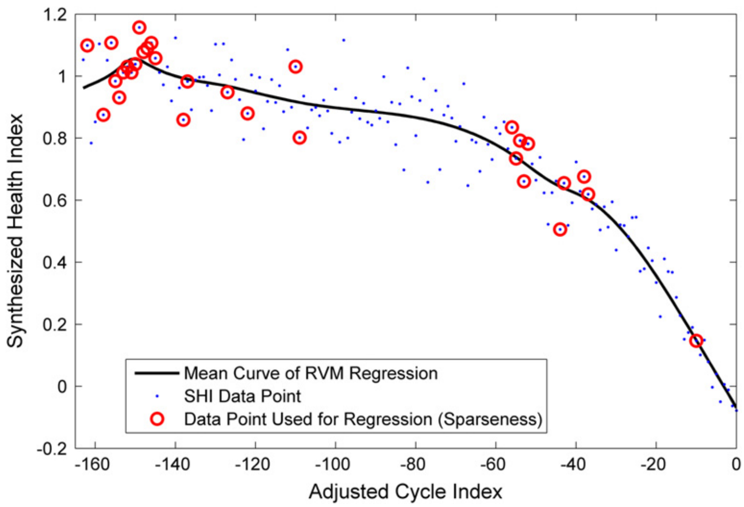

Fig. 7. Sparseness of the RVM regression.

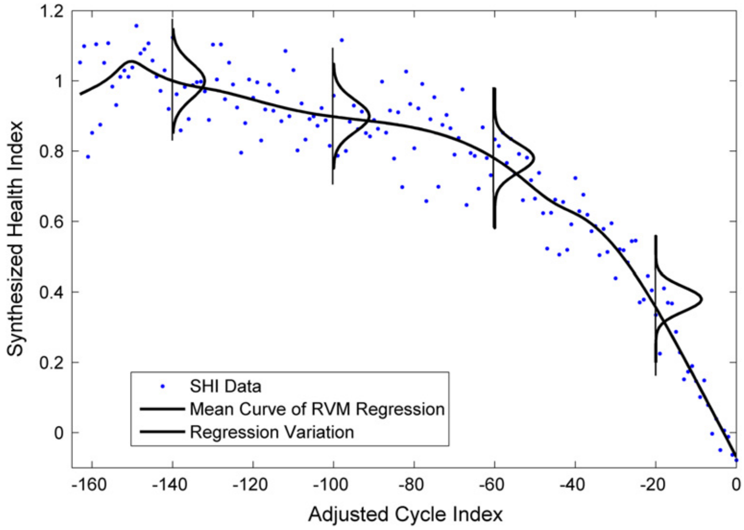

Fig. 8. SHI and the RVM regression.

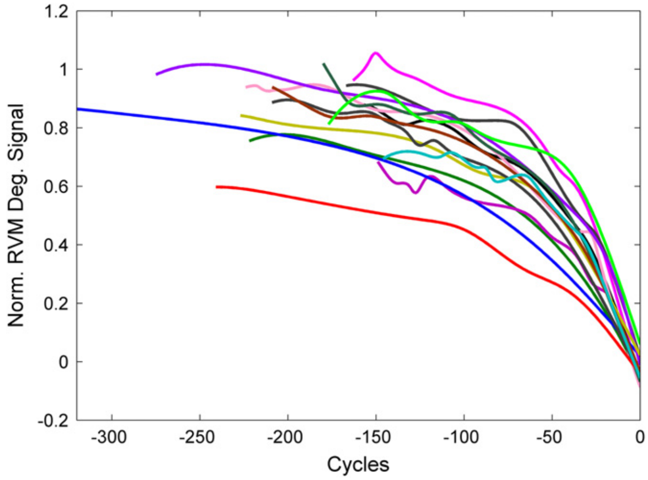

Fig. 9. Background degradation knowledge from SBL.

3.1.5. *Determination of initial health condition*

The online prediction process employed testing dataset obtained from 218 online system units. The adjusted cycle index was used to determine an initial health condition. As explained in Subsection 2.4, the optimization problem in Eq. (10) was solved to determine a time-scale initial health degradation state (T_0) with the testing dataset for an online engine unit while minimizing the sum-squared error between the online health data, $h(t_j)$, and predictive health degradation data, $h^p(t_j)$. Then, the predicted *RULs* and similarity weights of each online engine unit can be obtained using Eqs. (11) and (13) with $L=224$, $\Delta t=87$. Fig. 10 shows the process to determine the initial health degradation state (T_0) with the online testing data, $h(t_i)$, for the first engine unit and the predictive health degradation curve, $h^p(t)$, for the first unit. It should be noted that the offline learning process generates different predictive health degradation curves from K identical offline units. Repeating this process provided different projected *RUL*s (RUL_i for $i=1,\cdots,K$) on different predictive health degradation curves. The projected *RUL*s can be used to predict the *RUL* of an online unit in next section.

3.1.6. *Online RUL prediction*

From 218 offline engine units, the same number of the predictive health degradation curves and projected *RUL*s was obtained for each online engine unit. Likewise, the same number of similarity weights was sought for each online engine unit using Eq. (13). Eq. (12) modeled the *RUL* prediction for each online engine unit as a function of the projected *RUL*s while considering the first 50 largest similarity weights. Note that $h_i(t_i)$ and $h_i^p(t_i)$ are random as mentioned in Subsection 2.5. Thus, the similarity weights were modeled in a statistical manner, and so was the *RUL* of an online unit. Fig. 11 shows

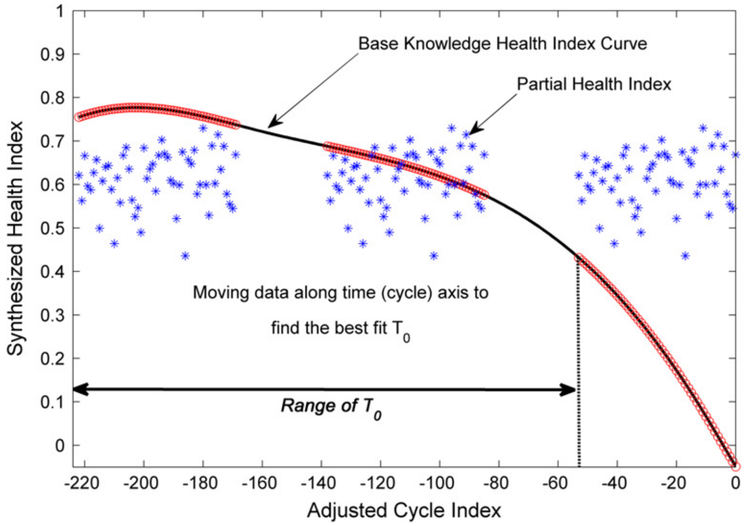

Fig. 10. Determination of initial health index.

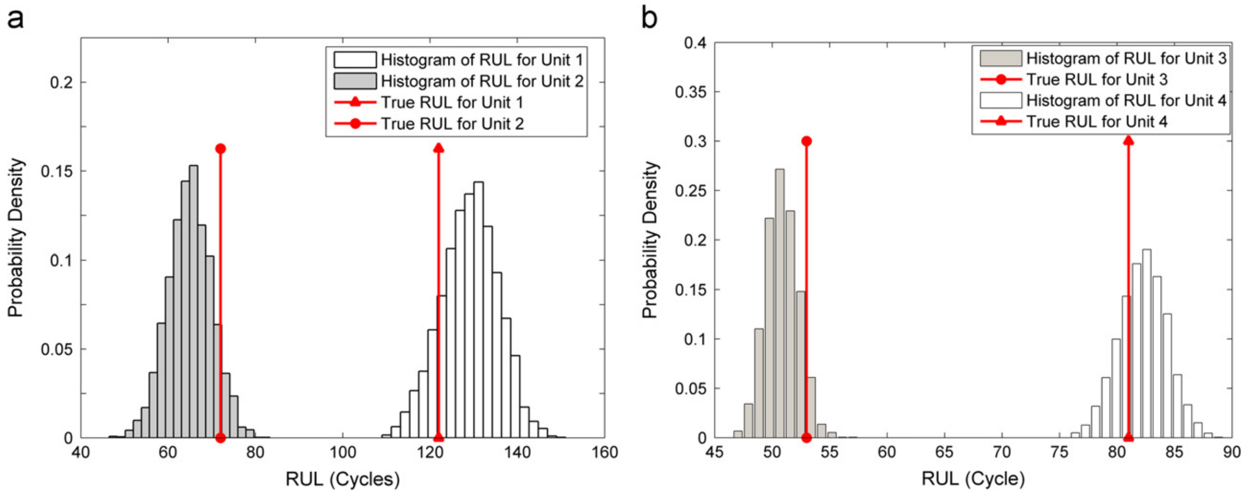

Fig. 11. Predicted *RUL* histograms with true RULs for (a) Units 1 and 2 and (b) Units 3 and 4.

the histograms of predicted *RULs* and the true RULs for the first four online engine units, in which 1000 random realizations were used to generate the histograms of the predicted RULs.

Using the mean value of the predictive RUL with the testing dataset, the cumulative score loss was then calculated using Eq. (15), which was used in the IEEE PHM challenge problem competition. We obtained an average score loss of 5.224 for the testing dataset:

$$d_k = \text{Predicted } RUL_k - \text{True } RUL_k$$
$$S_k = \begin{cases} e^{-d_k/13} - 1, d_k \le 0 \\ e^{-d_k/10} - 1, d_k \ge 0 \end{cases}$$
$$\text{Average Score Loss, } S = \frac{1}{K}\sum_{k=1}^{K} S_k \tag{15}$$

3.2. Electric cooling fan

In this section, we applied the generic prognostics framework to the health prognostics of electronic cooling fan units. Cooling fans are one of the most critical parts in system thermal solution of most electronic products and have been a major failure contributor to many electronic systems [35]. This study aims to demonstrate the proposed health prognostics methodology with 32 electronic cooling fans.

In this experimental study, thermocouples and accelerometers were used to measure temperature and vibration signals. To make time-to-failure testing affordable, the accelerated testing condition for the DC fan units was sought with inclusion of a small amount of tiny metal particles into ball bearings and an unbalanced weight on one of the fan units. The experiment block diagram of DC fan accelerated degradation test is shown in Fig. 12. As shown in the diagram, the DC fan units were tested with 12 V regulated power supply and three different signals were measured and stored in a PC through a data acquisition system. Fig. 13(a) shows the test fixture with 4 screws at each corner for the DC fan units. As shown in Fig. 13(b), an unbalanced weight was used and mounted on one blade for each fan. Sensors were installed at different parts of the fan, as shown in Fig. 14. In this study, three different signals were measured: the fan vibration signal from the accelerometer, the Printed Circuit Board (PCB) block voltage, and the temperature measured by the thermocouple. An accelerometer was mounted to the bottom of the fan with superglue, as shown in Fig. 14(a). Two wires were connected to the PCB block of the fan to measure the voltage between two fixed points, as shown in Fig. 14(b). As shown in Fig. 14(c), a thermocouple was attached to the bottom of the fan and measures the temperature signal of the fan. Vibration, voltage, and temperature signals were acquired by the data acquisition system and stored in PC. The data acquisition system from National Instruments Corp. (NI USB 6009) and the signal conditioner from PCB Group, Inc. (PCB 482A18) were used for the data acquisition purpose. In total, 32 DC fan units were tested at the same condition and all fan units run till failure.

The sensory signal screening found that the fan PCB block voltage and the fan temperature did not show clear degradation trend, whereas the vibration signal showed health degradation behavior. This study employed the root mean squares (RMS) of the vibration spectral responses at the first five resonance frequencies as the PHI for the DC fan

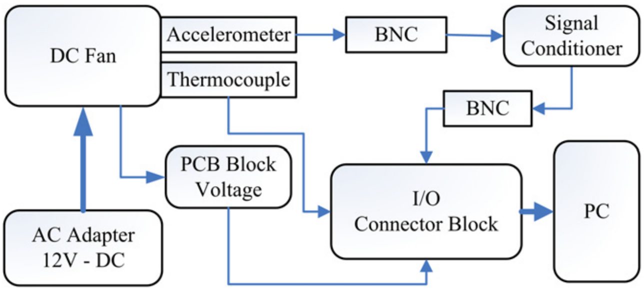

Fig. 12. DC fan degradation test block diagram.

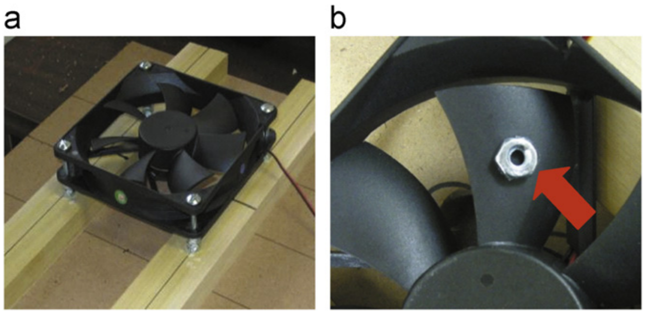

Fig. 13. DC fan test fixture (a) and the unbalance weight installation (b).

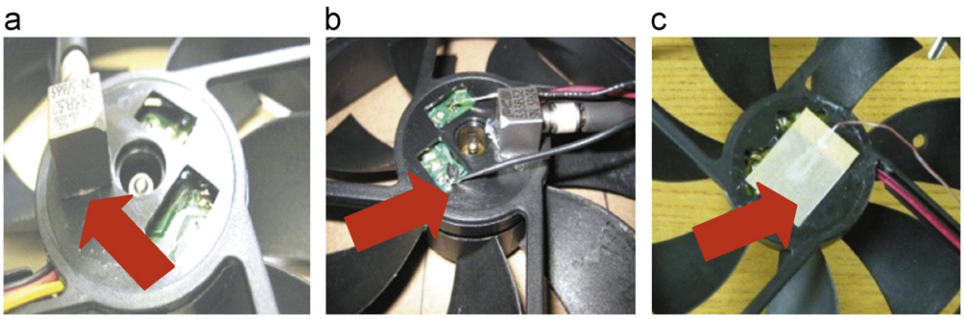

Fig. 14. Sensor installations for DC fan test, (a) accelerometer, (b) voltage measurement, and (c) thermocouples.

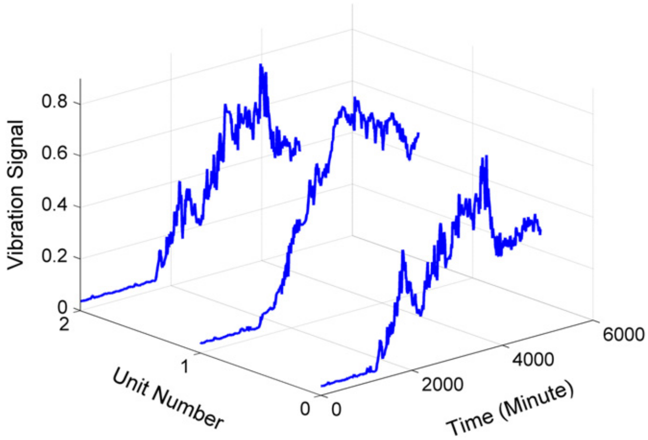

Fig. 15. Sample degradation signals from DC fan testing.

Table 5
Prognostics results for DC fans.

Predicted mean of RUL (min)				True life
Operation time, *T*	2000	3000	4000	
Test fan 1	2768	1802	1018	4957
Test fan 2	3615	2563	1394	5468
Test fan 3	3325	2298	1211	5124
Test fan 4	4107	2588	1662	5793
Error (%)	3.961	2.950	1.636	

prognostics. This selection was made based on a careful investigation on the degradation trends of different fans. Among 32 fan units, Fig. 15 shows the RMS signals of three fan units to demonstrate the health degradation behavior. The RMS signal gradually increased as the bearing in the fan degraded over time. It was found that the PHI is highly random and non-monotonic because of metal particles, sensory signal noise, and input voltage noise. We note that the combination of different vibration features (e.g., peak at different harmonic frequencies, and/or envelope analysis, etc.) could also be employed as multi-dimensional sensory data to construct the one-dimensional SHI for the health prognostics. However, we only employed the RMS data in our study simply because: (1) the degradation trend of the peak values at different harmonic frequencies exhibits significant similarity to the degradation trend of the RMS signal and (2) the aim of this case study was also to demonstrate the health prognostics for the PHI case.

For the DC fan prognostics, 28 fan units were randomly selected for the training dataset in the offline training process, while the rest were used as the testing dataset in the online prediction process. Following the same procedures of the previous case study, the prognostics work performed two distinguished processes: the offline training to obtain the predictive health degradation curves of the fan units using the RVM regression and the online prediction to predict and update the predictive *RUL*s of three online testing fan units using the SBI.

The *RUL* predictions for the three online testing fan units were conducted after 2000-, 3000-, and 4000-min uses and the results are shown in Table 5. The prediction results of the online testing fan units are quite accurate with the maximum error of 314 min of the fourth fan after 2000 min use. As more fan test data were used in the online prediction process, the prediction results become more accurate. The mean of the *RUL* prediction error after 4000 min use are much smaller than those after 2000 and 3000 min uses. As done in the previous case study, the final *RUL* prediction was conducted in a statistic manner. Fig. 16 shows the histogram of the predicted *RUL* for the first online testing fan after three different operation periods. Various statistical information of the predicted *RUL*, such as variation and confidence interval of prediction, can be extracted for condition-based maintenance.

4. Conclusion

This paper presented the probabilistic framework for structural health prognostics and uncertainty management. The proposed structural health prognostics methodology has the following three unique merits. Firstly, extended from the deterministic approach proposed by Wang et al. in Ref. [22], the probabilistic prognostics framework developed in this paper incorporates the uncertainty information into the RUL estimation and provides the predicted RULs with statistical

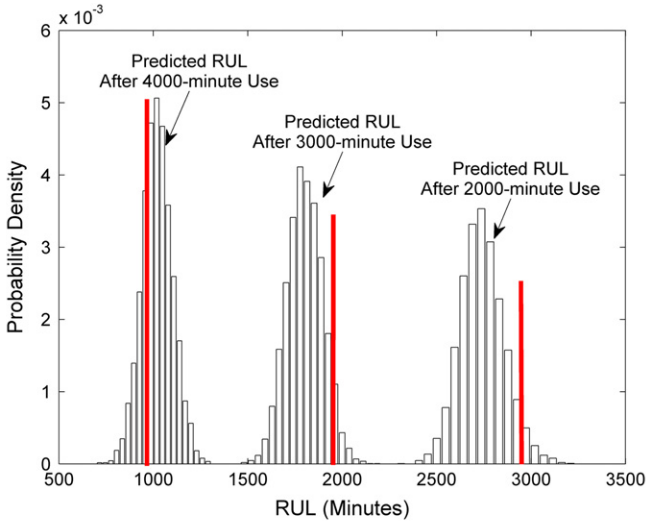

Fig. 16. Predicted *RUL* histogram for a DC fan.

bounds and distributions. The proposed approach can be generally applicable to structural health prognostics problems with run-to-failure training data. The proposed health indexes (PHI and SHI) provide a generic way to define the degree of health condition regardless of system complexity, sensory data size, physical data types, and so on. Secondly, the proposed approach employs the SBL technique using RVM in the offline training process to build predictive health degradation curves for offline training units in a statistical and sparse form. A set of predictive health degradation curves is treated as the background health knowledge which takes into account uncertainties in operational and manufacturing conditions. With this background health knowledge, the SBI technique was then proposed for predicting and continuously updating the *RUL* in the real-time online prediction process. Thirdly, the proposed prognostic uncertainty management framework provides a general way of propagating the uncertainties in the sensory signals through the prognostics process and enables the statistical prediction of RULs. Two engineering case studies (PHM challenge problem and the electric cooling fan prognostics problem) were used to demonstrate the effectiveness of the proposed unified data-driven prognostics methodology. Due to the generic capability of the proposed prognostics framework, its wide application to other engineered systems is promising. To further complete the probabilistic prognostic framework, future research could be pursued in multiple directions such as developing more comprehensive approaches to construct health indexes, to handle non-monotonic degradation curves, and to deal with lack of run-to-failure training units.

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