Journal of Intelligent Material Systems and Structures
http://jim.sagepub.com/

# A probabilistic detectability-based sensor network design method for system health monitoring and prognostics

Pingfeng Wang, Byeng D Youn, Chao Hu, Jong Moon Ha and Byungchul Jeon
*Journal of Intelligent Material Systems and Structures* published online 1 July 2014
DOI: 10.1177/1045389X14541496

Special Issue Article

Journal of Intelligent Material Systems and Structures

Journal of Intelligent Material Systems and Structures
1–12

DOI: 10.1177/1045389X14541496
jim.sagepub.com
SAGE

# A probabilistic detectability-based sensor network design method for system health monitoring and prognostics

**Pingfeng Wang[1], Byeng D Youn[2], Chao Hu[3], Jong Moon Ha[2] and Byungchul Jeon[2]**

**Abstract**

Significant technological advances in sensing promote the use of large sensor networks to monitor engineered systems, identify damages, and quantify damage levels. Prognostics and health management technique has been developed and applied for a variety of safety-critical engineered systems, given the critical needs of system health state awareness. The prognostics and health management performance highly relies on real-time sensory signals that convey system health–relevant information. Designing an optimal sensor network with high detectability of system health state is thus of great importance to the prognostics and health management performance. This article proposes a generic sensor network design framework using a detectability measure while accounting for uncertainties in material properties and geometric tolerances. Our contributions in this article are threefold: (1) the definition of a detectability measure to quantify the diagnostic/prognostic performance of a given sensor network, (2) the development of detectability analysis based on physics-based simulation and health state classification, and (3) the formulation of a generic sensor network design optimization problem as a mixed integer nonlinear programming. We employ the genetic algorithms to solve the sensor network design optimization problem. The merit of the proposed methodology is demonstrated with a power transformer system, which suffers from core and winding joint loosening due to consistent vibration.

**Keywords**

structural health monitoring, optimization, embedded intelligence

## Introduction

Significant technological advances in sensing and communication promote the use of large sensor networks (SNs) to monitor engineered systems, identify damages, and quantify damage levels. Prognostics and health management (PHM) techniques take full advantage of these advances and strive to enhance the safety and prolong the service lives of engineered systems through the means of in situ data acquisition, data feature extraction, and health diagnostics/prognostics to appropriately assess their health conditions and predict remaining useful lives (RULs). Through years of research efforts, PHM systems based on different types of sensors such as fiber optics, piezoelectric elements, and microelectromechanical system (MEMS) sensors have been developed for a wide variety of potential applications ranging from the civil, mechanical, and aerospace industries to automotive industry (Giurgiutiu et al., 2011; Li et al., 2004; Liu et al., 2012; Lonkar and Chang, 2013; Qiu et al., 2011). Despite the worldwide attention and significant advances in maturing the technologies for practical implementation, four primary challenges still remain in PHM: (1) sensing technologies to enhance sensitivity, repeatability, robustness, and reduce power consumptions of sensors; (2) communication techniques that allow connecting sensors with wired or wireless technology; (3) damage feature extraction research that focuses on the selection of damage

[1]Department of Industrial and Manufacturing Engineering, Wichita State University, Wichita, KS, USA
[2]Department of Mechanical and Aerospace Engineering, Seoul National University, Seoul, Korea
[3]Medtronic Inc., Minneapolis, MN, USA

**Corresponding author:**
Byeng D Youn, Department of Mechanical and Aerospace Engineering, Seoul National University, Seoul 151-742, Korea.
Email: bdyoun@snu.ac.kr

features that are usually tied to methods for sensor signal processing; and (4) damage pattern recognition and prognosis methods that enable recognizing the damage state of the engineered system and the severity of this damage (Chang and Markmiller, 2006). Although the literature in the structural health monitoring has employed different methods such as damage probability density plots (Flynn et al., 2011; Zhao et al., 2007) to quantify the structural damages based on particular sensor outputs, it is clear that successful accomplishment of a system health diagnostics/prognostics mission relies extremely on an effectively designed SNs, as stated in the first challenge. Thus, one of the most important tasks in PHM system is the development of a generic SN design framework that takes into consideration different system failure mechanisms, sensor characteristics, as well as a variety of uncertainties involved.

Most of the research activities for SN design in the past decade targeted on maximizing the coverage and minimizing the power consumption of SNs (Buczak et al., 2001; Chakrabarty et al., 2002) for various applications that require the data acquisition. Research on the optimal sensor allocation has been driven by the need of optimizing large SNs for efficient monitoring activities. Several methods have been developed to enhance the detection efficiency and minimize the uncertainty in decision-making based on data acquired from the SNs. In the work by Field and Grigoriu (2006), the optimum SN identified by the number of sensors, location, and the range of sensors is determined based on methods from decision theory to enhance the tracking and identification of vehicles for the purpose of surveillance, in which the uncertainties related to the assumptions made regarding traffic composition and possible vehicle trajectories are considered for the SN design. Guratzsch and Mahadevan (2006) also defined the optimum SN for system health monitoring under uncertainties as the sensor placement optimization (SPO) problem that can maximize the probability of damage detection, where uncertainties related to structure simulation using finite element (FE) models and other are modeled as Gaussian random field variables. Furthermore, Li et al. (2006) introduced the concept of the modal participation factor from structural dynamics as an extension of the mode shape summation plot to obtain a vector of sensor placement indices based on the weighted components of the mode shape matrix corresponding to the sensor positions. However, the presented work is based on the widely used norm-based sensor placement method where uncertainty nature of structures has not been explored. Ntotsios et al. (2006) presented an approach to address the stochastic nature of the sensor measurements by introducing the information entropy to measure the performance of a sensor configuration and employing asymptotic estimates to justify the selection of optimal sensor configurations based on nominal structural modes so that the time history details of measurement data could be ignored. Azarbayejani et al. (2008) employed an artificial neural network approach to identify the optimum sensor placement for a bridge case study. The sensor allocation problem is handled within the context of uncertainty and information entropy. A Bayesian method is used to quantify damage in the structure based on the change in modal information and the information entropy is then used to compute a scalar measure of uncertainty in the structural damage features. A heuristic sequential sensor placement algorithm is then used to predict the optimal sensor configuration. Flynn and Todd (2010) also employed a Bayesian method for optimal sensor placement with active sensing using guided ultrasonic waves, in which a global optimality criterion is derived so that the optimal configuration problem can be established as an optimization problem to minimize the expected total presence of either type I or type II error during the damage detection process. Works by Ntotsios et al. (2006), Udwadia (1994), and Heredia-Zavoni and Esteva (1998) showed the importance of addressing the issue of uncertainty in handling the optimal sensor configuration. Other researchers (Kirkegaard and Brincker, 1994; Papadimitriou et al., 2000) also reported the use of the information entropy and information functions such as the Fisher information to formulate the objective function for optimal sensor allocations. The aforementioned approaches showed the significance of considering uncertainties introduced by sensor units, engineered systems, as well as the operation conditions in the SN design problem and presented unique methods to deal with uncertainties in the damage detection. However, most of these methods were developed for the problem of distributing a finite set of sensors to detect a specific type of system damage, and their applications are tied to and largely restricted by the type of failure mechanisms under consideration.

Given the significance of an SN for the PHM and years of research efforts, the design of SNs nonetheless becomes tied to the system damage feature of choice, and the development of a generic design methodology is still a hurdle to overcome. Thus, this article presents a probabilistic framework for the SN design optimization for system health monitoring and prognostics, in which several technical contributions have been made. First, detectability is defined as a unified quantitative measure of SN performance in a probabilistic form. This detectability measure is unique in that it indicates the SN performance on accurate detection of system health states while considering uncertainty in manufacturing and operation processes; second, a detectability analysis method is developed based on the computer simulation and health state classification. Third, the SN design optimization is formulated as a mixed integer nonlinear programming (MINLP) problem based on the defined detectability measure, and the genetic algorithms (GAs) are used as an optimizer in this study. The developed SN design methodology will be demonstrated with a power transformer case study. The rest of this article is organized in the following way. Section "Detectability-based SN design framework" will present the proposed SN design optimization framework, while section "SN design against power transformer mechanical joint failure" will present the results of the power transformer case study. The conclusion of the work will be given in section "Conclusion."

## Detectability-based SN design framework

This section presents the detectability-based SN design framework for system health monitoring and prognostics. The first subsection defines a detectability measure in a probabilistic manner that indicates the diagnostic/prognostic performance of a given SN while accounting for uncertainty in manufacturing and operation processes. Subsequently, the detectability analysis and its procedure will be presented. Finally, the detectability-based SN design optimization framework is discussed.

### *Detectability measure of an SN*

Due to uncertainty in manufacturing and system operation processes, the physical behaviors (e.g. vibrational responses) of an engineered system that can be captured by SNs are heavily random. Thus, the SN performance on accurate detection of system health states must be defined in a probabilistic manner. In the proposed SN design framework, a set of health states must be first classified based on historical failure data and/or expert knowledge, for example, a rolling bearing's health state can be defined as normal, nick, scratches, more nicks, and failure (Zhang et al., 2005). The correct detection rate of each health state will then be defined as one of the SN performance measures for the purpose of health diagnostics and prognostics. This correct detection rate can be formulated as a conditional probability that an SN can detect the same health state as that at which the system is operated. On the contrary, the incorrect detection rate can be formulated as a conditional probability that an SN provides incorrect health state information. These correct and incorrect detection rates can constitute the probability-of-detection (PoD) matrix for a given SN design, from which the SN detectability can be derived.

*PoD matrix.* A general form of the PoD matrix for a given SN with a number of health states (i.e. $HS_i$, $i = 1, 2, \ldots, N_{HS}$) is shown in Table 1, where one element $P_{ij}$ is defined as the conditional probability that the system is detected to be operated at $HS_j$ by the SN given that the system is operated at $HS_i$. Clearly, $P_{ij}$ represents the probabilistic relationship between the true system health state and the health state detected by the SNs. Mathematically, it is expressed as

$$P_{ij} = \Pr(\text{Detected as } HS_j | \text{System being at } HS_i) \qquad (1)$$

By the definition, the $i$th diagonal term in the PoD matrix represents a conditional probability of correct detection for the $i$th health state.

*Detectability measure.* Based on the PoD matrix, the detectability of the $i$th system health state $HS_i$ is defined as

$$D_i = P_{ii} = \Pr(\text{Detected as } HS_i | \text{System is at } HS_i) \qquad (2)$$

The above definition provides a probabilistic measure for the diagnostic/prognostic performance of an SN while considering uncertainty in manufacturing and system operation processes. The diagonal terms in the PoD matrix, which represent the probabilities of correct detection for predefined health states, will determine the overall SN detection performance. With the predefined detectability requirements, these diagonal terms in the PoD matrix will then constitute $N_{HS}$ number detectability constraints in the SN design process. Since these detectability constraints involve the computation of multiple conditional probabilities, an efficient and accurate methodology for detectability analysis must be developed.

### *Detectability analysis*

This section presents a detectability analysis method based on computer simulation and system health state classification. Sensory signals indicate different system health states (e.g. healthy or failure) through the differences of system physical responses captured by an SN. Thus, a valid simulation model that can precisely predict the overall trend of the physical responses of the system is required for detectability analysis. Verification and validation (V&V) is a primary means of assessing the accuracy of simulation models and is of great importance for SN design based on computer simulation (Guratzsch and Mahadevan, 2006). Thorough discussion of V&V is beyond the scope of this study, and more information can be found from Youn et al. (2011), ASME (2006), Oberkampf et al. (2004), and Babuska and Oden (2004). In this study, we assume that all numerical models used in this article are valid and can provide accurate simulation results compared with actual systems. In the rest of this subsection, a mathematical example will be used first to derive valuable information of detectability evaluation, and the detectability analysis method will then be presented.

**Table 1.** Probability of detection (PoD) matrix.

| Probability | | Detected health state | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | … | $N_{HS}$ |
| True health state | 1 | $P_{11}$ | $P_{12}$ | … | $P_{1N_{HS}}$ |
| | 2 | $P_{21}$ | $P_{22}$ | … | $P_{2N_{HS}}$ |
| | … | … | … | … | … |
| | $N_{HS}$ | $P_{N_{HS}1}$ | $P_{N_{HS}2}$ | … | $P_{N_{HS}N_{HS}}$ |

PoD: probability of detection.

*A mathematical example.* Suppose that only one sensor will be used for damage detection. It is assumed that the sensor output from a system with the healthy condition (Health State 1, $HS_1$) follows a normal distribution as $N\ (0, 0.5^2)$. The sensor output from the system with a minor damage (Health State 2, $HS_2$) can be characterized as $N\ (1, 0.8^2)$, whereas the sensor output from the system with a major damage follows a normal distribution as $N\ (5, 1^2)$. In what follows, we will find out the detectability values for all three defined health states based on the available information.

To calculate the detectability value for each health state, it is necessary to classify any given set of testing sensory data into one of the three health states. This can be accomplished simply by defining a normalized distance, the $z$-score for this example, between the testing data and the sensor output distribution for each health state. The normalized distance measures the divergence of the testing data from the sensor output distribution of each health state, and consequently, the testing data should be classified into the health state which has the smallest normalized distance. For any two health states, there could exist one neutral point that leads to an equal normalized distance to two neighboring health states. In this example, the neutral point $X_{1-2}$ between two health states, $HS_1$ and $HS_2$, can be calculated as

$$\frac{X_{1-2} - 0}{0.5} = \frac{1 - X_{1-2}}{0.8} \tag{3}$$

which provides $X_{1-2} = 0.3846$. Similarly, the neutral point $X_{2-3}$ between $HS_2$ and $HS_3$ can be calculated as

$$\frac{X_{2-3} - 1}{0.8} = \frac{5 - X_{2-3}}{1} \tag{4}$$

which provides $X_{2-3} = 2.7778$. Figure 1 shows the sensor output distributions for three health states and the neural points that separate these health states.

Based on the detectability definition in equation (1) and the neutral points calculated in equations (3) and (4), the detectability values for this mathematical example can be evaluated as

$$\begin{aligned} D_1 &= P_{11} = \Pr(\text{Detected as } HS_1 | \text{System is at } HS_1) \\ &= \Pr\left(X \leq X_{1-2} | X \sim N(0, 0.5^2)\right) \\ &= 0.7791 \end{aligned} \tag{5}$$

$$\begin{aligned} D_2 &= P_{22} = \Pr(\text{Detected as } HS_2 | \text{System is at } HS_2) \\ &= \Pr\left(X_{1-2} \leq X \leq X_{2-3} | X \sim N(1, 0.8^2)\right) \\ &= 0.7660 \end{aligned} \tag{6}$$

$$\begin{aligned} D_3 &= P_{33} = \Pr(\text{Detected as } HS_3 | \text{System is at } HS_3) \\ &= \Pr\left(X \geq X_{2-3} | X \sim N(5, 1^2)\right) \\ &= 0.9869 \end{aligned} \tag{7}$$

From the analytical evaluation of the detectability in the mathematical example above, it is clear that classification of the health states and statistical distributions of sensor outputs are crucial for the SN detectability analysis. However, in most engineering applications, an SN is composed of multiple sensors and required to deal with much more than three different health states. Consequently, the analytical analysis of SN detectability through the calculation of neutral points between health states becomes practically impossible. Besides, the statistical distributions of sensors' outputs for all different health states as assumed in the mathematical example are usually not available. Instead, a finite set of sensory data will be used as training data set to characterize the uncertainties of sensor output for each system health state. Thus, more sophisticated health state classification tools (Sohn et al., 2003) such as linear discriminant analysis (Mosavi et al., 2012), support vector

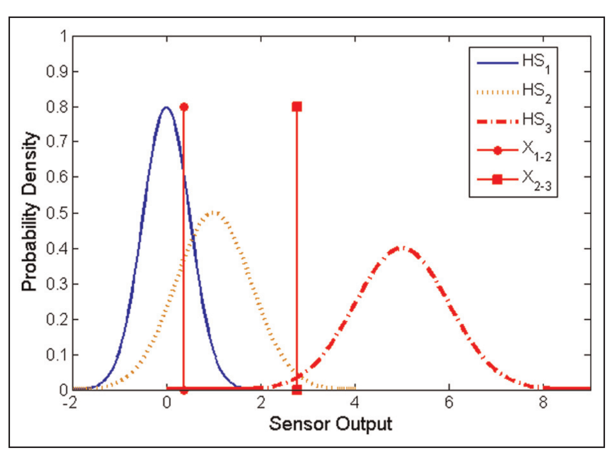


**Figure 1.** Sensor outputs and neutral points between health states.
HS: health state.

**Table 2.** Training data sets for three health states.

| HS | Sensor 1 | Sensor 2 |
|---|---|---|
| $HS_1$ | $\sigma$*rand (1, *n*) − 1.4 | $\sigma$*rand (1, *n*) |
| $HS_2$ | $\sigma$*rand (1, *n*) − 3 | $\sigma$*rand (1, *n*) + 0.7 |
| $HS_3$ | $\sigma$*rand (1, *n*) | $\sigma$*rand (1, *n*) − 0.5 |

MD: Mahalanobis distance; HS: health state.

machine (Kim et al., 2013), and Mahalanobis distance (MD) (Nguyen et al., 2014; Niu et al., 2011), which should be able to classify any given set of multi-dimensional testing data into one of multiple different health states based on a finite set of training data, are needed for the SN detectability analysis. In this study, the MD classifier that can be effectively used for this classification purpose is employed.

*MD classifier for HS classification.* The MD provides a powerful method of measuring how similar one set of testing data is with another predefined set of training data and can be very useful for identifying which predefined health state is the most similar one to the system health state represented by the testing data. For the purpose of health state classification, the MD classifier quantitatively measures the similarity between a given testing sensory data set and the training data set for the $i$th system health state through the MD, expressed as

$$MD_i = (X - M_i)^T \Sigma^{-1} (X - M_i) \tag{8}$$

where $X$ is the given testing sensory data set to be classified, $M_i$ is the vector of mean values of the training data set for the health state $HS_i$, and $\sum$ is the covariance matrix of the training data set for $HS_i$. The testing sensory data set will be classified by the classifier into a predefined system health state that gives the smallest MD value, or in other words the highest similarity. The following mathematical example demonstrates the system health state classification using the MD classifier. The $\sigma$ value used in this mathematical example is 0.5.

In this example, two sensors are used and three system health states including one healthy state, $HS_1$, and two faulty states, $HS_2$ and $HS_3$, are predefined. There are 10 sets of sensory data for each health state as the training data sets (i.e. $n = 10$), as shown in Table 2. To demonstrate the MD classifier for health state classification, five sets of testing data, as shown in the first two columns of Table 3, need to be classified into one of the three predefined health states. Using the MD classifier, the MD values for each testing data set can be calculated with the training data sets shown in Table 2 using equation (8). The MD values together with the classified system health state for each sensory data set are also shown in Table 3.

Based on the above procedure, the PoD matrix as defined in Table 1 can be evaluated. Suppose that there are totally $T_i$ sets of testing data from the health state $HS_i$, and within which $T_{ij}$ sets are classified into the health state $HS_j$ by the MD classifier, where $i, j = 1, 2, \ldots, N_{HS}$, the element $P_{ij}$ in the PoD matrix can be approximately calculated based on the definition as

$$T_{ij} = \begin{pmatrix} 70 & 3 & 2 \\ 5 & 70 & 0 \\ 32 & 0 & 43 \end{pmatrix} \tag{9}$$

$$P_{ij} \approx \frac{T_{ij}}{T_i} \tag{10}$$

$$P_{ij} = \begin{pmatrix} 0.93 & 0.04 & 0.03 \\ 0.07 & 0.93 & 0 \\ 0.43 & 0 & 0.57 \end{pmatrix} \tag{11}$$

Since any set of testing data from the health state $HS_i$ will definitely be classified into one of the predefined $N_{HS}$ health states, the following equation regarding $P_{ij}$ can be obtained

$$\sum_{j=1}^{N_{HS}} P_{ij} = 1 \tag{12}$$

$$\sum_{j=1}^{N_{HS}} P_{ij} = 0.93 + 0.04 + 0.03 = 1 \tag{13}$$

The above equation (12) suggests that the summation of each row in the PoD matrix will always equal to 0.

**Table 3.** System health state classification using MD classifier.

| Sensory data | | MD | | | Classified state |
|---|---|---|---|---|---|
| $S_1$ | $S_2$ | $HS_1$ | $HS_2$ | $HS_3$ | |
| −1.66 | 0.13 | 0.39 | 10.44 | 15.76 | $HS_1$ |
| −2.26 | 0.89 | 5.72 | 2.53 | 33.60 | $HS_2$ |
| −0.96 | 0.95 | 4.25 | 19.88 | 8.90 | $HS_1$ |
| −2.48 | −0.31 | 6.27 | 6.08 | 34.94 | $HS_1$ |
| 0.09 | −1.64 | 18.75 | 77.73 | 9.34 | $HS_3$ |

MD: Mahalanobis distance; HS: health state.

**Table 4.** Procedure for detectability analysis.

| | |
|---|---|
| Step 1 | Definition of health states—to define the system health states based on experts' knowledge or historical data |
| Step 2 | Sensory data acquisition—to collect training and testing data sets for each predefined system health state for a given SN design |
| Step 3 | Extract a subset of training and testing data, for a given SN design, from the data sets collected in Step 2, in which only the data from sensors in the given SN design will be remained |
| Step 4 | Health classification—to perform classification using the MD classifier defined by equation (8) for the extracted subset of the training and testing data |
| Step 5 | Detectability calculation—to calculate the detectability measure for all health states using equation (14) |

SN: sensor network; MD: Mahalanobis distance.

Similarly, the detectability, diagonal terms in the PoD matrix, for the health state $HS_i$ can be obtained as

$$D_i = P_{ii} \approx \frac{T_{ii}}{T_i} \tag{14}$$

$$D_i = (\,0.93 \quad 0.93 \quad 0.57\,) \tag{15}$$

*Procedure of detectability analysis.* The overall procedure of the detectability analysis is summarized in Table 4. As mentioned in the preceding discussion, the definition of the system health states enables the performance evaluation of a candidate SN and should be treated as a crucial step for the SN design. Through defining multiple system health states, SNs can be designed to tackle multiple failure mechanisms and modes for engineered systems. After defining the health states, collecting training and testing data sets for all health states is the next step, which can be accomplished through valid computer simulation models, such as finite element analysis (FEA) or structural dynamic analysis. The size of the training and testing data sets will determine the accuracy of the detectability evaluation using the proposed MD classifier. With the training and testing data sets available, the detectability for each predefined health state for a given SN design can be evaluated in the same way as we did in the previous example.

## SN design optimization

Appropriate selection of sensing devices, such as fiber optic sensors, piezoelectric sensors, MEMS sensors, accelerometers, or acoustic sensors, is determined by the sensor's specifications, such as full-scale dynamic range, sensitivity, noise floor, and analog-to-digital converter resolution. Thus, the design variables involved in the proposed SN design framework are the decision variables for the selection of sensing devices, numbers of selected sensing devices, their locations, and the parameters for controlling the sensing process, such as sampling frequency, sampling period, and power configuration. The design constraints are detectability requirements considering uncertainty presented in manufacturing and system operation processes. With all factors considered above, the SN design optimization problem can be formulated as

$$\begin{aligned} &\text{Minimize} \quad \mathrm{C}(\mathbf{X_T}, \mathbf{X_N}) \\ &\text{subject to} \quad D_i(\mathbf{X_T}, \mathbf{X_N}, \mathbf{X_{Loc}}, \mathbf{X_s}) \geq D_i^t \\ &\qquad\qquad (i = 1, 2, \ldots, N_{HS}) \end{aligned} \tag{16}$$

where C is the cost involved and it is calculated as the product of the number of sensors and the sum of sensor material and installation costs, $\mathbf{X_T}$ is a vector of the binary decision variables for the selection of the types of sensing devices, $\mathbf{X_N}$ is a vector consisting of numbers of each selected type of sensing devices, $\mathbf{X_{Loc}}$ is a three-dimensional (3D) vector of the location of each sensing device, and $\mathbf{X_s}$ is a vector of sensing control parameters; $N_{HS}$ is the total number of predefined health states for the engineered system. $D_i$ is the detectability of the SN for the $i$th predefined health state, which is a function of the design variables $\mathbf{X_T}$, $\mathbf{X_N}$, $\mathbf{X_{Loc}}$, and $\mathbf{X_s}$, whereas $D_i^t$ is the target SN detectability for the $i$th predefined health state. It is noted that the formulation of the SN design optimization problem bears a resemblance to that of the reliability-based design optimization problem (Youn et al., 2006; Youn and Xi, 2008) with the exception that the former uses the detectability as the constraint and the latter uses the reliability as the constraint.

The SN design optimization problem in equation (16) contains discrete decision variables for the selection of sensing devices, integer variables for the number of selected sensing devices, as well as continuous variables for the sensor locations. Thus, it is formulated as an MINLP problem (Adjiman et al., 2000), and heuristic algorithms such as GAs can be used as the optimizer for the optimization purpose. In this study, the GA is employed for the example problem that will be detailed in the subsequent section. More alternative algorithms for solving the MINLP problem can be found in Adjiman et al. (2000) and Wei and Realff (2004).

Figure 2 shows the flowchart of the SN design optimization process. As shown in this figure, the process starts from an initial SN design and goes into the design optimization subroutine (the right-hand side gray box), which will carry out the SN cost analysis, call the

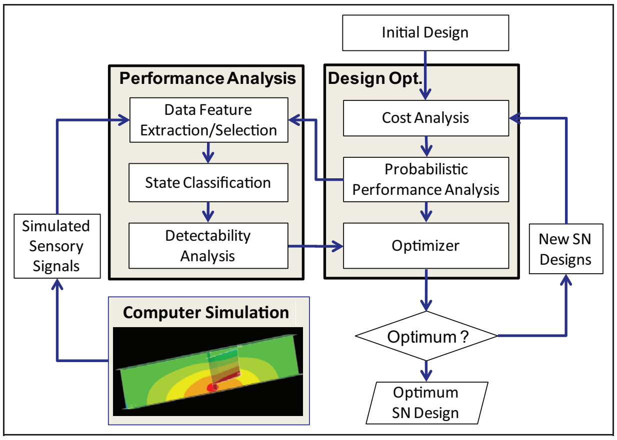


**Figure 2.** Flowchart of detectability-based SN design for system health monitoring and prognostics.
SN: sensor network.

performance analysis subroutine (the left-hand side gray box) to evaluate the performance of the SN at the current design, and execute the optimizer to generate the new SN design if the optimality condition is not met. In the performance analysis subroutine, the detectability analysis as discussed in the previous section will be carried out. Before solving the optimization problem, valid system simulation models have to be built, and computer simulations have to be accomplished so that the training and testing data sets for each predefined health state are available.

## SN design against power transformer mechanical joint failure

Power transformers are among the most expensive elements of high-voltage power systems. Health monitoring of power transformers enables the transition from traditional schedule-based maintenance to condition-based maintenance, resulting in significant reductions in operation and maintenance costs (Leibfried, 1998). Due to the difficulties of direct measurement inside the transformer, the data that are most often used for both diagnosis and prognosis of the transformers are obtained through indirect measurements (Rivera et al., 2000). For example, temperature measurements were first accomplished at accessible points, and modeling the gradient information can then be used to induce the temperature peaks in some areas; electric parameters and analysis of moisture content of the cooling oil are often performed for the diagnosis and condition-based maintenance of transformers, with frequency response analysis of electric characteristics being common (Allan et al., 1992); the vibrations of the magnetic core and the windings could characterize transitory overloads and permanent failures before any irreparable damage occurs. This case study aims at designing an optimum SN on the front wall surface of a power transformer. The measurements of the transformer vibration responses induced by the magnetic field loading enable the detection of mechanical failures of winding support joints inside the transformer.

### Description of the case study

In this study, the winding support joint loosening is considered as the failure mode, the detection of which will be realized by collecting the vibration signal, induced by the magnetic field loading with a fixed frequency on the power transformer core, using the optimally designed SN at the external surface of the transformer. The FE model of a power transformer was created in ANSYS 10 as shown in Figure 3, where one exterior wall is concealed to make the interior structure visible. Figure 4 shows 12 simplified winding support joints with 4 for each winding. The transformer is fixed at the bottom surface, and a vibration load with the frequency of 120 Hz is applied to the transformer core. The joint loosening was realized by reducing the stiffness of the joint itself. Different combinations of the loosening joints will be treated as different health

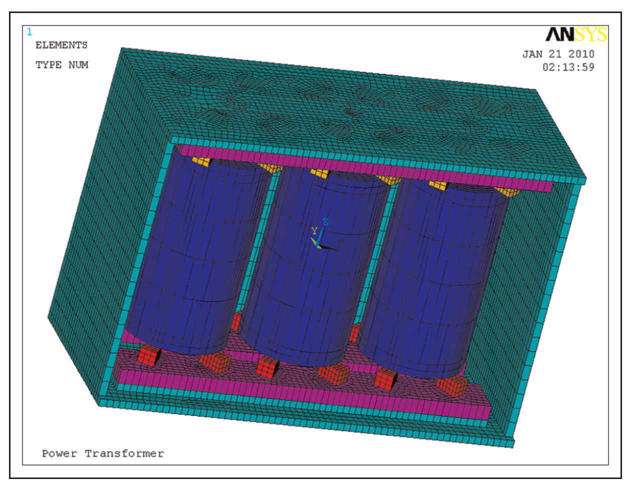


**Figure 3.** A power transformer FE model (without the covering wall).
FE: finite element.

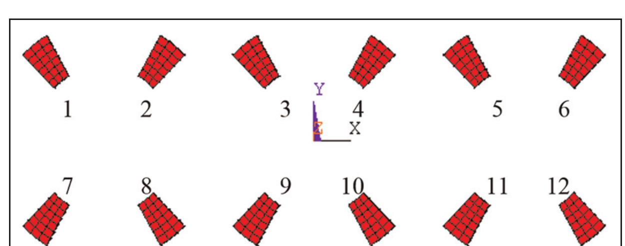


**Figure 4.** Winding support joints and their numberings.

**Table 5.** Random property of the power transformer.

| Random variable | Physical meaning | Randomness (cm, g, degree) |
|---|---|---|
| $X_1$ | Wall thickness | $N$ (3, $0.06^2$) |
| $X_2$ | Angular width of support joints | $N$ (15, $0.3^2$) |
| $X_3$ | Height of support joints | $N$ (6, $0.12^2$) |
| $X_4$ | Young's modulus of support joint | $N$ (2e12, $4\text{e}10^2$) |
| $X_5$ | Young's modulus of loosening joints | $N$ (2e10, $4\text{e}8^2$) |
| $X_6$ | Young's modulus of winding | $N$ (1.28e12, $3\text{e}10^2$) |
| $X_7$ | Poisson's ratio of joints | $N$ (0.27, $0.0054^2$) |
| $X_8$ | Poisson's ratio of winding | $N$ (0.34, $0.0068^2$) |
| $X_9$ | Density of joints | $N$ (7.85, $0.157^2$) |
| $X_{10}$ | Density of windings | $N$ (8.96, $0.179^2$) |

**Table 6.** Definition of system health states.

| Health state | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Loosening joints | – | 1 | 2 | 3 | 1, 2 | 1, 3 | 1, 5 | 1, 9 | 1, 11 |

states of the power transformer which will be detailed in the next subsection.

The uncertainties in this case study are modeled as random parameters with corresponding statistical distributions listed in Table 5, which includes the material properties, such as Young's modulus, densities, and Poisson's ratios, for support joints and windings, as well other parts in the power transformer system. Besides, the geometry parameters are also considered as random variables. In this study, the random variables are considered to be normally distributed. It is worth noting that the material properties and geometric tolerances could be distributed in different ways other than just normal. However, it is expected that the effect of the non-normally distributed random variables on the SN design result could be minimal. For probabilistic design with uncertainties involving different types of distributions, the readers are directed to Youn et al. (2003), Wang and Wang (2013), and Youn and Wang (2007). These uncertainties will be propagated into the structural vibration responses and will be accounted for when designing an optimum SN.

## Health states and simulations

For the purpose of demonstrating the proposed SN design methodology, nine representative health states (see Table 6) were selected from all possible combinations of 12 winding support joint failures. Among these nine selected health states, $HS_1$ denotes the healthy condition without any loosening joint, whereas $HS_2$–$HS_9$ are health states with either one or two loosening joints. According to the statistical properties of random parameters in Table 5, 200 sets of random samples were generated and the simulations for each of nine health states were carried out. For example, for the simulation of $HS_1$, a power transformer model with no loosing joints was simulated 200 times based on the generated random samples, while a power transformer model with joint 1 loosening (a much lower Young's modulus, $X_5$, is assigned to joint 1) was simulated 200 times for $HS_2$. The vibration amplitudes for all the FE nodes on the outer wall surfaces were saved as the simulation results for each health state. Among the 200 simulated data sets for each health state, 100 sets were used as the training data set and the others were used as the testing data set that was used to evaluate the SN detectability. The stress contour of the healthy state power transformer at the nominal values of the random parameters from the structural simulation is shown in Figure 5, whereas the vibration response of the covering wall is shown in Figure 6. The vibration amplitude of each node on the surface of the covering wall was used as the simulated output of one sensor (accelerometer).

As mentioned in the previous section, this case study problem is formulated as designing the SN on the surface of the covering wall of the power transformer to minimize the cost of the SN while satisfying the detectability constraints for each health state, that is, the detectability should be greater than a target detectability of 0.95. The design variables in this case study include the following: (1) the total number of accelerometers, (2) the locations of the accelerometers, and (3) the sensing orientation ($X$ or $Z$) of each accelerometer.

## Results and discussion

Following the flowchart shown in Figure 2 and the detectability analysis procedure listed in Table 4, the SN design problem in this case study was solved using the GA. Figure 7 shows the detectability for each of nine health states at the optimum SN design while

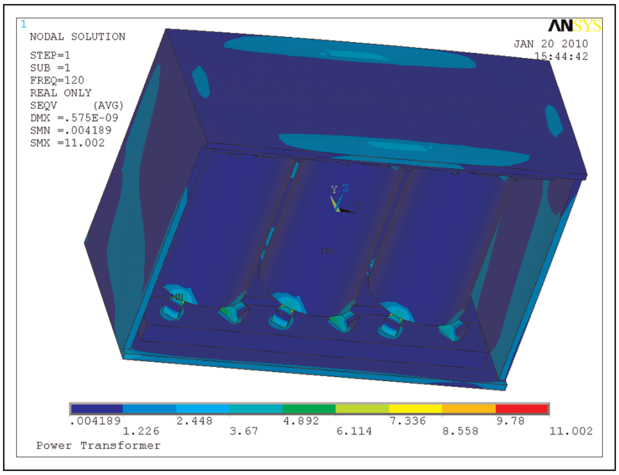


**Figure 5.** Stress contour of the winding supports for the healthy state power transformer.

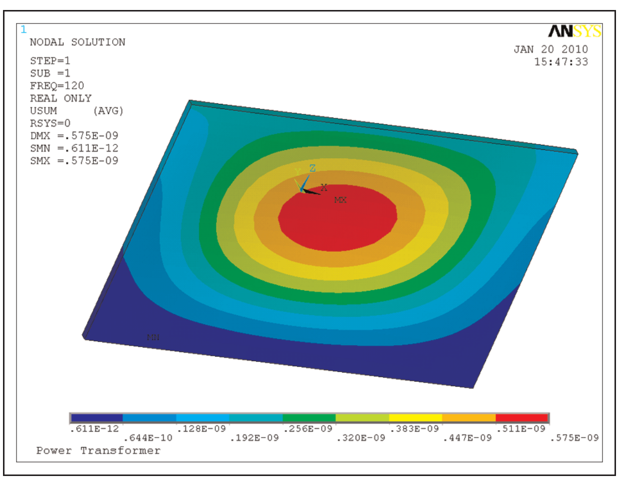


**Figure 6.** Vibration displacement contour of the covering wall for the healthy state power transformer.

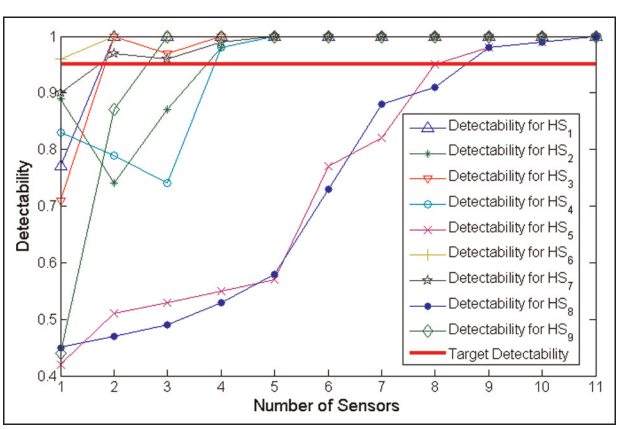


**Figure 7.** Minimum detectability at optimum design versus total number of sensors.
HS: health state.

**Table 7.** Optimum SN design for power transform case study.

| Sensor index | Location (cm) | | Direction |
|---|---|---|---|
| | x | z | |
| 1 | −56.4 | 0.0 | Z |
| 2 | 67.2 | −34.4 | X |
| 3 | −2.6 | −30.0 | Z |
| 4 | 49.7 | −34.4 | X |
| 5 | −57.9 | 30.0 | X |
| 6 | −30.6 | 15.3 | X |
| 7 | 27.5 | 30.0 | X |
| 8 | 39.3 | 35.2 | X |
| 9 | 59.1 | 0.0 | X |

SN: sensor network.

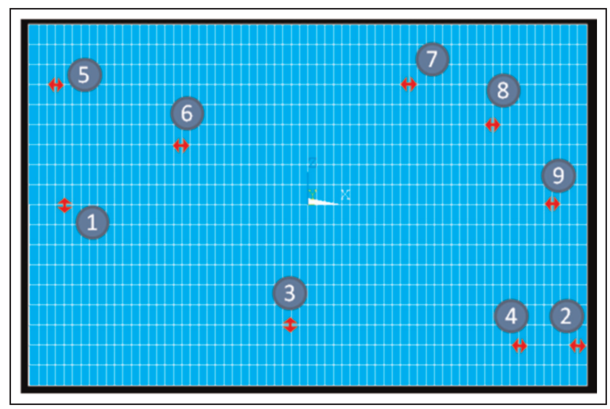


**Figure 8.** Optimal design of the distributed SN for power transformer case study.
SN: sensor network.

stepwisely increasing the number of the sensors. With the target detectability being 0.95, we obtained the optimum SN design on the outer wall surface (140 cm × 90 cm) with totally nine sensors, as shown in Table 7 and Figure 8.

The results of this case study suggest that the proposed SN design framework could be used to tackle the SN design problems for complicated engineering systems with multiple system health states considering system input uncertainties. Several important remarks regarding the results of the case study are presented as follows.

*Remark 1.* In this study, the MD has been used as a method for the detectability evaluation. It is worth noting that there are different metrics, such as the Kullback–Leibler divergence (Perez-Cruz, 2008) and others, that could be used to identify the similarity between the training data and the testing data. It would be very interesting to explore different metrics in future continuous study of this work.

*Remark 2.* The GA was implemented for the design optimization and repeatedly executed for 10,000 times. Although, for most of times, the optimization converged to the optimal design, the convergence to local minima was also observed. Improvement of the computational robustness will help improve the performance of the proposed SN design methodology. Thus, it would be interesting to investigate other optimization algorithms (e.g. the particle swarm optimization (del Valle et al., 2008)) to make the SN design process more robust.

*Remark 3.* Due the computational time, only 100 samples were simulated for each health state, resulting in two decimal digits of precision in the detectability estimates. In order to obtain results with higher precision, we need more samples from the computer simulation.

*Remark 4.* We also note that to make the designed SN more reliable, we can integrate the redundancy of sensors to the proposed framework simply by treating it as an additional set of design variables and the SN reliability as an additional constraint.

*Remark 5.* The proposed approach intends to develop an SN based on a set of predefined health states, and the results thus depend on the quality of the information used to define these health states. In practical applications, the discrete health states could be defined based on different failure modes of interest for a target structural system, whereby the information for the health state definition could come from risk analysis and/or failure mode effect analysis (FMEA). Moreover, the information can be acquired from structural simulation, field history, and/or expert opinions. In the case when discrete health states are not available, different levels of severity of structural damages (such as magnitude of fatigue crack lengths) could be used as a potential way to transform the continuous health state to different discrete ones. In this study, the information used for health state definition is assumed to be of high quality, although presented with uncertainties, and future study could be conducted to explore how the quality of information (such as inclusion of sensing noise or opinion bias between different experts) might impact the SN design results.

*Remark 6.* We acknowledge that a valid simulation model with high predictive capability is essential to deriving an effective SN design and that an invalid computer model could possibly lead to a meaningless design. In this case study, however, we have not provided an experimental justification of the model validity since the validation of a simulation is not the focal point of this study. Indeed, our intent here is to set up an engineering problem that provides some empirical evidence as to the merits of our proposed methodology. As has been discussed, our initial results in this case study point to the effectiveness of our proposed approach. We still note that to help circumvent potential problems associated with the model validity, we plan on providing comprehensive experimental justification of our design as our future study.

## Conclusion

This article presented a probabilistic framework for SN design optimization using a detectability measure while accounting for uncertainty in manufacturing and system operation processes. The proposed work consists of three major technical contributions. First, we defined a probabilistic detectability measure to quantify the performance of a given SN on detecting the system health states in a statistical manner. Second, based on the computer simulation and health state classification, we developed the detectability analysis method, where the MD classifier was employed for the health state classification. In case multiple health measures or complicated configuration of the classification boundary are engaged, a more advanced classification method, that is, support vector machine, can be possibly used. Third, we formulated the SN design framework as an MINLP. The GA was used as the optimizer to solve the SN design optimization problem. The power transformer case study demonstrated that the proposed SN design framework is feasible to handle multiple system health states considering input uncertainties involved such as material properties and geometric tolerances.

### Declaration of conflicting interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

### Funding

This work was partially supported by a grant from the Energy Technology Development Program (2010101010027B) and International Collaborative R&D Program (0420-2011-0161) of Korea Institute of Energy Technology Evaluation and Planning (KETEP), funded by the Korean government's Ministry of Trade, Industry and Energy the Basic Research Project of Korea Institute of Machinery and Materials (Project Code: SC0830) supported by a grant from Korea Research Council for Industrial Science & Technology; and the Institute of Advanced Machinery and Design at Seoul National University (SNU IAMD).

### References

Adjiman CS, Androulakis IP and Floudas CA (2000) Global optimization of mixed-integer nonlinear problems. *AIChE Journal* 46(9): 1769–1797.

Allan D, Blundell M, Boyd K, et al. (1992) New techniques for monitoring the insulation quality of in-service HV

apparatus. *IEEE Transactions on Electrical Insulation* 27(3): 578–585.
ASME (2006) *Guide for Verification and Validation in Computational Solid Mechanics*. New York: American Society of Mechanical Engineers.
Azarbayejani M, El-Osery AI, Choi KK, et al. (2008) A probabilistic approach for optimal sensor allocation in structural health monitoring. *Smart Materials and Structures* 17(5): 055019.
Babuska I and Oden JT (2004) Verification and validation in computational engineering and science: basic concepts. *Computer Methods in Applied Mechanics and Engineering* 193(36–38): 4057–4066.
Buczak AL, Wang H, Darabi H, et al. (2001) Genetic algorithm convergence study for sensor network optimization. *Information Sciences* 133(3–4): 267–282.
Chakrabarty K, Iyengar SS, Qi H, et al. (2002) Grid coverage for surveillance and target location in distributed sensor networks. *IEEE Transactions on Computers* 51(12): 1448–1453.
Chang F-K and Markmiller JFC (2006) A new look in design of intelligent structures with SHM. In: *Proceedings of 3rd European workshop: structural health monitoring*, Granada, 5–7 July, pp. 5–20. Lancaster, PA: DEStech Publications, Inc.
del Valle Y, Venayagamoorthy GK, Mohagheghi S, et al. (2008) Particle swarm optimization: basic concepts, variants and applications in power systems. *IEEE Transactions on Evolutionary Computation* 12(2): 171–195.
Field RV and Grigoriu M (2006) Optimal design of sensor networks for vehicle detection, classification, and monitoring. *Probabilistic Engineering Mechanics* 21(4): 305–316.
Flynn EB and Todd MD (2010) A Bayesian approach to optimal sensor placement for structural health monitoring with application to active sensing. *Mechanical Systems and Signal Processing* 24(4): 891–903.
Flynn EB, Todd MD, Wilcox PD, et al. (2011) Maximum-likelihood estimation of damage location in guided-wave structural health monitoring. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 467(2133): 2575–2596.
Giurgiutiu V, Lin B, Santoni-Bottai G, et al. (2011) Space application of piezoelectric wafer active sensors for structural health monitoring. *Journal of Intelligent Material Systems and Structures* 22(12): 1359–1370.
Guratzsch RF and Mahadevan S (2006) Sensor placement design for SHM under uncertainty. In: *Proceedings of 3rd European workshop: structural health monitoring*, Granada, 5–7 July, pp. 1168–1175. Lancaster, PA: DEStech Publications, Inc.
Heredia-Zavoni E and Esteva L (1998) Optimal instrumentation of uncertain structural systems subject to earthquake ground motions. *Earthquake Engineering & Structural Dynamics* 27(4): 343–362.
Kim Y, Chong JW, Chon KH, et al. (2013) Wavelet-based AR–SVM for health monitoring of smart structures. *Smart Materials and Structures* 22(1): 015003.
Kirkegaard PH and Brincker R (1994) On the optimal location of sensors for parametric identification of linear structural systems. *Mechanical Systems and Signal Processing* 8(6): 639–647.
Leibfried T (1998) Online monitors keep transformers in service. *IEEE Computer Applications in Power* 11(3): 36–42.
Li DS, Li HN and Fritzen CP (2006) On the physical significance of the norm based sensor placement method. In: *Proceedings of 3rd European workshop: structural health monitoring*, Granada, 5–7 July, pp. 1160–1167. Lancaster, PA: DEStech Publications, Inc.
Li H-N, Li D-S and Song G-B (2004) Recent applications of fiber optic sensors to health monitoring in civil engineering. *Engineering Structures* 26(11): 1647–1657.
Liu L, Liu S and Yuan F-G (2012) Damage localization using a power-efficient distributed on-board signal processing algorithm in a wireless sensor network. *Smart Materials and Structures* 21(2): 25005.
Lonkar K and Chang F-K (2013) Modeling of piezo-induced ultrasonic wave propagation in composite structures using layered solid spectral element. *Structural Health Monitoring* 13(1): 50–67.
Mosavi AA, Dickey D, Seracino R, et al. (2012) Identifying damage locations under ambient vibrations utilizing vector autoregressive models and Mahalanobis distances. *Mechanical Systems and Signal Processing* 26: 254–267.
Nguyen T, Chan T and Thambiratnam D (2014) Controlled Monte Carlo data generation for statistical damage identification employing Mahalanobis squared distance. *Structural Health Monitoring*. Epub ahead of print 10 February. DOI: 10.1177/1475921714521270 (12 pp.).
Niu G, Singh S, Holland SW, et al. (2011) Health monitoring of electronic products based on Mahalanobis distance and Weibull decision metrics. *Microelectronics Reliability* 51(2): 279–284.
Ntotsios E, Christodoulou K and Papadimitriou C (2006) Optimal sensor location methodology for structural identification and damage detection. In: *Proceedings of 3rd European workshop: structural health monitoring*, Granada, 5–7 July, pp. 1160–1167. Lancaster, PA: DEStech Publications, Inc.
Oberkampf WL, Trucano TG and Hirsch C (2004) Verification, validation, and predictive capability in computational engineering and physics. *Applied Mechanics Reviews* 57(5): 345–384.
Papadimitriou C, Beck JL and Au S-K (2000) Entropy-based optimal sensor location for structural model updating. *Journal of Vibration and Control* 6(5): 781–800.
Perez-Cruz F (2008) Kullback-Leibler divergence estimation of continuous distributions. In: *2008 IEEE international symposium on information theory*, Toronto, ON, Canada, 6–11 July, pp. 1666–1670. New York: IEEE.
Qiu L, Yuan S, Zhang X, et al. (2011) A time reversal focusing based impact imaging method and its evaluation on complex composite structures. *Smart Materials and Structures* 20(10): 105014.
Rivera HL, Garcia-Souto JA and Sanz J (2000) Measurements of mechanical vibrations at magnetic cores of power transformers with fiber-optic interferometric intrinsic sensor. *IEEE Journal of Selected Topics in Quantum Electronics* 6(5): 788–797.
Sohn H, Farrar CR, Hemez FM, et al. (2003) *A review of structural health monitoring literature: 1996–2001*. Report no. LA-13976-MS, February. Los Alamos, NM: Los Alamos National Laboratory.

Udwadia FE (1994) Methodology for optimum sensor locations for parameter identification in dynamic systems. *Journal of Engineering Mechanics: ASCE* 120(2): 368–390.
Wang Z and Wang P (2013) A maximum confidence enhancement based sequential sampling scheme for simulation-based design. *Journal of Mechanical Design* 136(2): 021006.
Wei J and Realff MJ (2004) Sample average approximation methods for stochastic MINLPs. *Computers & Chemical Engineering* 28(3): 333–346.
Youn BD and Wang P (2007) Bayesian reliability-based design optimization using eigenvector dimension reduction (EDR) method. *Structural and Multidisciplinary Optimization* 36(2): 107–123.
Youn BD and Xi Z (2008) Reliability-based robust design optimization using the eigenvector dimension reduction (EDR) method. *Structural and Multidisciplinary Optimization* 37(5): 475–492.
Youn BD, Choi KK and Park YH (2003) Hybrid analysis method for reliability-based design optimization. *Journal of Mechanical Design* 125(2): 221.
Youn BD, Choi KK, Du L, et al. (2006) Integration of possibility-based optimization and robust design for epistemic uncertainty. *Journal of Mechanical Design* 129(8): 876–882.
Youn BD, Jung BC, Xi Z, et al. (2011) A hierarchical framework for statistical model calibration in engineering product development. *Computer Methods in Applied Mechanics and Engineering* 200(13–16): 1421–1431.
Zhang X, Xu R, Kwan C, et al. (2005) An integrated approach to bearing fault diagnostics and prognostics. In: *Proceedings of the 2005 American control conference*, 8–10 June, pp. 2750–2755. New York: IEEE.
Zhao X, Gao H, Zhang G, et al. (2007) Active health monitoring of an aircraft wing with embedded piezoelectric sensor/actuator network: I. Defect detection, localization and growth monitoring. *Smart Materials and Structures* 16(4): 1208.